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Hidden Markov Model to Optimize Coordination Relationship for Learning Behaviour

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Abstract

School communities interact dynamically, much like the agents in a multi-agent system. For coordinated action, relationships between agents in a multi-agent system must be handled. One technique for persuading individuals to behave in a coordinated manner is to manage the role of agents in generating knowledge, attitudes, and practices. Managing these connections is difficult due to the large number of unknowns. Modeling can aid in the clarification of agent relationships. Coordination mechanisms can be modeled using Markov models. Agents can demonstrate and consider how their actions affect other agents in order to achieve desired behavior goals. This paper extends the state space of Partially Observable Markov Decision Processes (POMDPs) with an agent model to make them multi-agent friendly.

Keywords: *Hidden Markov Model, Optimization, Coordination Relationship, Learning Behavior*

INTRODUCTION

Since the mid-1950s, the Markov model has been used in psychology to infer cognitive states from data sequences in learning experiments (Miller, 1952; Steiner and Greeno, 1969). This is accomplished through learning experiments. It is now widely acknowledged as a useful tool for integrating massive sets of longitudinal observations (Langeheine, Stern, and Van de Pol, 1994) on topics ranging from implicit learning (Visser, Raijmakers, & van der Maas, 2009) to well-being (Eid and Langeheine, 2007). They have also been used in the classroom to investigate actor agreements (Weingart, 1999), peer scaffolding that occurs from interactions between students in a synchronous networked setting (Pata, Lehtinen, and Sarapuu, 2006), and to compare the efficacy of students' various counseling strategies (Duys, and Headrick, 2004). A number of sophisticated models were also developed to shed light on the sequential decision-making process (Fu and Anderson, 2006; Niv, 2009).

Within the scope of this study, it is assumed that there are personnel who can assist the work unit's operations in its pursuit of the goal. As a result, one can assert that there is such a thing as an autonomous agent. An autonomous agent is an intelligent creature that acts rationally and deliberately in relation to its goal and the information it currently possesses (Wooldridge, 1999). The focus of the field of research known as multi-agent systems, or MAS for short, is the study of autonomous agents interacting in the same environment, sharing resources, bargaining and collaborating to achieve their goals, forming coalitions, and experiencing disputes, among other things.

If autonomous agents in a multi-agent system are to complete their missions successfully, there must be sufficient coordination (MAS). Such coordination is required to handle the various types of dependencies that naturally arise when agents have goals that are related to one another, when they share the same environment, or when resources are shared among multiple parties. Coordination is the process by which an agent considers his own actions as well as the actions that he anticipates others will take. The purpose of this process is to ensure that the community operates in a consistent manner (Jennings, 1996). One of the most important issues to consider when developing coordination approaches for multi-agent systems is how to manage instances in which the actions of one agent affect the operations of the actions of another agent (Malone and Crowston,

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1994). For example, one agent's actions may make it easier for another agent to carry out their own actions, or they may allow another agent to carry out a different activity.

This paper discusses the challenge of representing and administering coordination mechanisms between work units in an unpredictable environment. The goal is to instill consistent behavior. We argue that in order for an agent to behave coherently, they must be able to infer and explain coordinating relationships. An agent working toward a goal, for example, must be able to anticipate potential conflicts and determine whether the actions of another agent will help or hinder their efforts to achieve the goal.

Partially observable Markov decision processes, also known as interactively partially observable Markov decision processes, can provide a framework for sequential decision making in a partially observable multiagent environment (I-POMDP). They modified the POMDP algorithm to allow it to be used in a multiagent context by including a computed model of other agents in the state space alongside the physical environment's state. In Bayesian games, a model is related to a type because it contains all of the information that determines the behavior of agents, such as their preferences, abilities, and beliefs. I-POMDP takes a subjective approach to studying strategic behavior, based on a decision theory framework that takes into account a decision maker's perspective in interaction. This enables a more nuanced analysis of the phenomenon under investigation.

RELATED WORKS

Black and Wiliam (1998) compiled a collection of studies that support the conclusion that instructors who use assessment to drive instructional decision making have better quantifiable outcomes than teachers who do not use assessment (Black and Wiliam, 2010). This mathematical model of decision-making requires two parts: an assessment model and an instructional activity model. Both of these models are required. The evidence-centered assessment design (ECD) method is a principled method for generating mathematical models for instructional components, but it does not account for instructional impacts (Mislevy, Steinberg, and Almond, 2003).

A critical component of this profession is the development of qualitative models for the effects of education that teachers, tutors, and other instructors use in their reasoning. Each unit describes the subjects covered by the training's consequences as well as the prerequisite criteria that ultimately lead to success. Quantitative information, such as how likely a student is to succeed in the lesson (regardless of whether or not the prerequisites have been completed), and the magnitude of the effect if the lesson is successful, is typically lacking (or unsuccessful).

Consider a music tutor who works with a student one-on-one to teach them how to play a musical instrument and meets with them on a weekly basis. The weekly tutoring session consists of assessing the students' progress and assigning new assignments for the following week. To make the model easier to understand, assume that the majority of the learning occurs during the first week of the student's practice. Tutors can show students new ideas and methods, but they won't truly understand them until they put them into practice for at least a week.

The tutor will present practice activities for the student to work on at each meeting between the student and the tutor, which will typically include a combination of different types of exercises as well as songs (music or a significant portion of a piece of music). There is no denying that Vygotsky’s zone theory of proximal development applies to this option (Vygotsky and Cole, 1978). If the tutor does not teach in a challenging enough manner for the student, the student will not gain much from the experience. If the tutor's lessons are too difficult for the student, very little learning will take place. A portion of the challenge is determining current levels of proficiency in order to provide appropriate instruction. Assume that the tutor can adjust the level of difficulty of the exercise along two dimensions: mechanics and fluency. In this article, $a_t = (a_{t,\text{ mechanics}}, a_{t,\text{ fluency}})$ refers to the activities performed by the instructor (or the duties assigned by the tutor) at the time indicated by t (Li et al., 2019).

Lessons take place at various times, $t_1, t_2, t_3, \ldots$. This will happen at regular intervals for the most part, but there may be gaps in between (vacation, missed lessons, etc.). Because the events that occur between lessons are frequently interesting, the time remaining until the next lesson is abbreviated as $\Delta t_n = t_{n+1} - t_n$. The notation, however, accounts for missed lessons, vacations, and other circumstances that may result in uneven spacing. This is due to the fact that Δt_n will remain constant over time in many applications. Even with a small number of examples, a significant portion of the problem's characteristics must be represented.

MATERIALS AND METHODS

POMDP

A Markov model is a strategy that analyzes the behavior of many variables based on their current state in order to predict how they will behave in the future. A starting method is required to get started with this option. POMDP (Partially Observed Markov Decision Process) was used in this study, and its complexity was later reduced to I-POMDP (Interactive-Partially Observed Markov Decision Process). In order to find solutions to management challenges, both approaches examine the behavior of prior agents, beginning with the individual belief process and working their way up to an action. If POMDP is intended to work with a single agent, I-POMDP is intended to work with multiple agents and is an excellent choice for school communities with a variety of components.

To compute the single agent model, first derive POMDP, which is defined in the following sentence (Boutilier, Dean, and Hanks, 1999; Hauskrecht, 2000; Kaelbling, Littman, and Cassandra, 1998; Monahan, 1982).

***POMDPi* = ⟨*S*, *Ai*, *Ti*, Ω*i*, *Oi*, *Ri*⟩**

Where:

S = the range of existing environmental conditions

Ai = sequence of actions agent i can perform

Ti = transition function –$Ti : S \times Ai \times S \rightarrow [0, 1]$ which describes the result of the action of agent i.

Ω*i* = series of observations made by agent i.

Oi = agent observation function – $Oi : S \times Ai \times \Omega i \rightarrow [0, 1]$ which is the observation probability if the agent performs various actions that cause changes in conditions or conditions that are different from the previous one.

Ri = reward function that represents the characteristics of agent i. ($Ri: S \times Ai \rightarrow \mathbf{R}$)

RESULTS AND DISCUSSION

In this research we introduce variables, such as:

Knowledge

Attitude

Practice

The role of Headmaster ($S_{\Omega A}$)

Teachers' role ($S_{\Omega A}$)

Parents' role ($S_{\Omega A}$)

School committee's role ($S_{\Omega A}$)

Agents' role and students' KAP along with school community in North Sumatra Province can be found as written from the following table.

Table 1. Level of School Community Coordination Mechanism

No	Description	Persentage
1	Head Masters' role	37,77
2	Teachers' role	40,12
3	School committee' role	42,15
4	Parents' role	48,08
5	Students knowledge	25,96
6	Students attitude	55,49
7	Students practice	65,94

Level of School Community Coordination Mechanism in determining Students KAP can be determined using the following diagram:

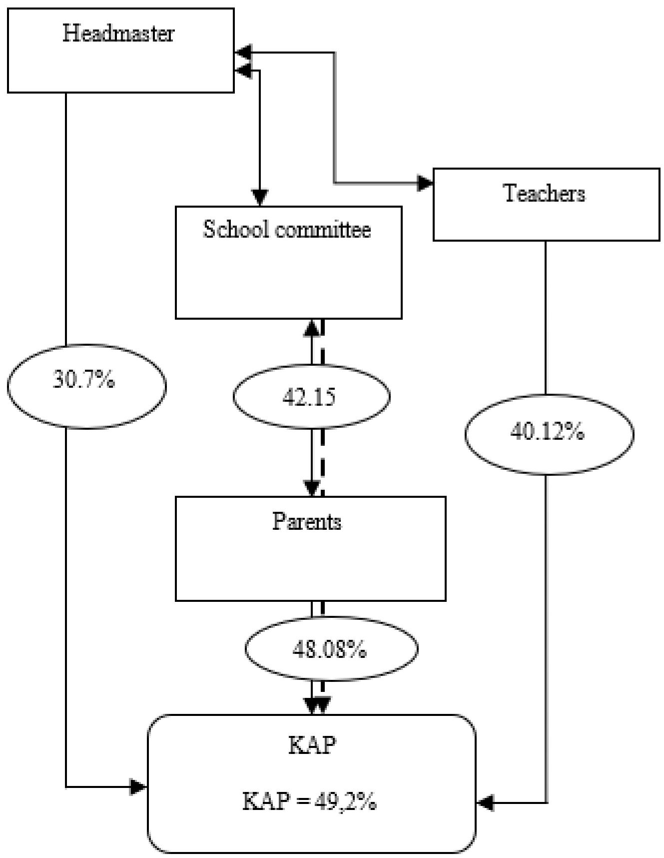

Figure 1. Level of school community coordination mechanism

Model Development

MAS is assumed to be the same as a school community due to the fact that the school system is also a dynamic system, consisting of separate parts and with relationships and interactions within them.

Create a School Community Coordination Diagram developed from Influence Diagrams and Theory of Learning

Enter data from the distribution ($S_{\Omega A}$)

Calculate the value of probabilities for change in the school community.

The Markov model is a method for analyzing the current behavior of several variables in order to predict the behavior of these variables in the future.

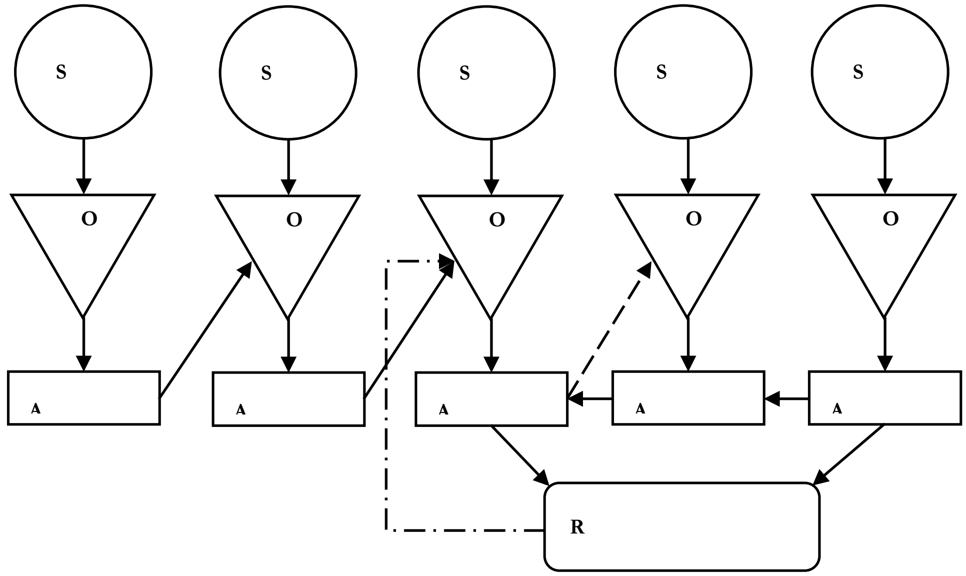

Figure 2. Model of influence diagram and distribution SOA KOM SEKOLAH

In this paper we use partially observed Markov Decision Process (POMDP) approach to get the probabilities value for the transition of the students' KAP (Boutilier, Dean, and Hanks, 1999; Hauskrecht, 2000; Kaelbling, Littman, and Cassandra, 1998; Monahan, 1982).

In state i the partially observed Markov decision Process defined as $POMDP_i = \langle S_i, A_i, T_i, \Omega_i, O_i, R_i \rangle$.

Where S_i is a sequence of existing environmental conditions.

A_i is a sequence of action of agent i can perform.

T_i is transition function $-T_i: S \times A_i \times S \rightarrow [0,1]$ which describes the results of agent i action.

Ω_i is a sequence of observations performed by agent i.

O_i is observation function of agent $-O_i: S \times A_i \times \Omega_i \rightarrow [0,1]$ which is the probability of observation if the agent takes various actions that cause changes in conditions or conditions that are different from before.

R_i is a reward function that represents a characteristic of agent i $(R_i: S \times A_i \rightarrow R)$.

Table 1. Distribution of data based on Variable S, Ω, and A

School Community	Agents' Role				Students' Kap			KAP Average
Variable	Head Master	Teachers	School Committee	Parents	K	A	P	
S	46.77	32.00	34.85	47.81	48.42	27.93	51.76	42.70
Ω_i	29.64	39.71	52.80	37.08	31.00	27.74	13.11	23.95
A_i	23.59	28.29	12.35	15.11	20.57	44.33	35.12	33.34

From the influence diagram it can be seen that it is necessary to calculate the probability provided that the observation has occurred so that the probability value that you want to calculate is the conditional probability value.

$$P(A_{kom}|\Omega_{kom})$$

Solve it, then the result can be written as:

$$P(A_{kom}|\Omega_{kom}) = \frac{P(A_{kom} \cap \Omega_{kom})}{P(\Omega_{kom})}$$

But in the diagram, the occurrence of Ω (observation) can only occur after the occurrence of S (condition), so it can be written as:

$$P(A_{kom}|\mathrm{S}_{kom})$$

From conditional probability, we get:

$$P(A_{kom}|\mathrm{S}_{kom}) = \frac{P(A_{kom} \cap \mathrm{S}_{kom})}{P(\mathrm{S}_{kom})}$$

$$P(A_{kom} \cap \mathrm{S}_{kom}) = \mathrm{S}_{kom} \times \Omega_{kom} = 0.349 \times 0.528$$

$$P(\mathrm{S}_{kom}) = 0.349$$

$$P(\Omega_{kom}|\mathrm{S}_{kom}) = 0.528$$

Then for,

$$P(\mathrm{A}_{kom}|\Omega_{kom}) = \frac{P(A_{kom} \cap \Omega_{kom})}{P(\Omega_{kom})}$$

$$= \frac{P(A_{kom} \cap \Omega_{kom})}{P(\Omega_{kom}|\mathrm{S}_{kom})}$$

$$P(\mathrm{A}_{kom} \cap \Omega_{kom}) = \mathrm{S}_{kom} \times A_{kom} \times \Omega_{kom}$$

$$= 0.349 \times 0.124 \times 0.528$$

Therefore,

$$P(\mathrm{A}_{kom}|\Omega_{kom}) = 0.124 \times 0.349 = 0.043$$

$$P(R \cap A_S) = S_S \times A_S = 0.427 \times 0.333 = 0.14$$

$$P(A_S) = P\left(A_S\middle|\Omega_S, A_g\right) = 0.08$$

$$P(R \cap A_{kS}) = S_{kS} \times A_{kS} = 0.468 \times 0.236 = 0.11$$

$$P(A_{kS}) = P(A_{kS}|\Omega_{kS}) = 0.273$$

Thus,

$$P(R|A_S, A_{kS}) = \left(\frac{0.08}{0.14}\right)\left(\frac{0.11}{0.273}\right)$$

$$= 0.57 \times 0.403$$

$$= 0.23$$

$$TR_iK = (S_i \times \Omega_i \times A_i)_i \times TR_K$$

$$TR_iA = (S_i \times \Omega_i \times A_i)_i \times TR_A$$

$$TR_iP = (S_i \times \Omega_i \times A_i)_i \times TR_P$$

Simulation results for the development of students' KAP due to the change of School community agents, will be given as follows.

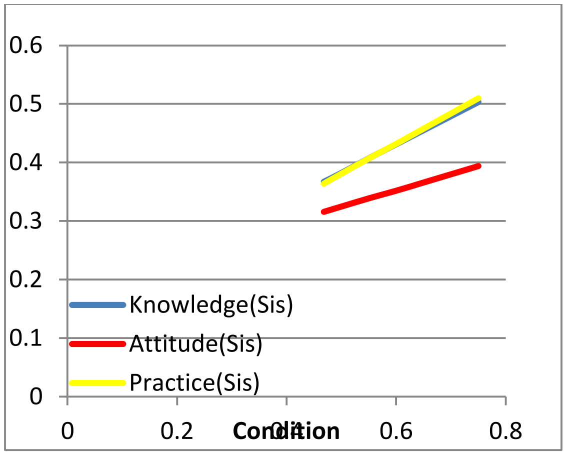

Figure 3. Development of students' KAP due to the change of Headmaster Condition

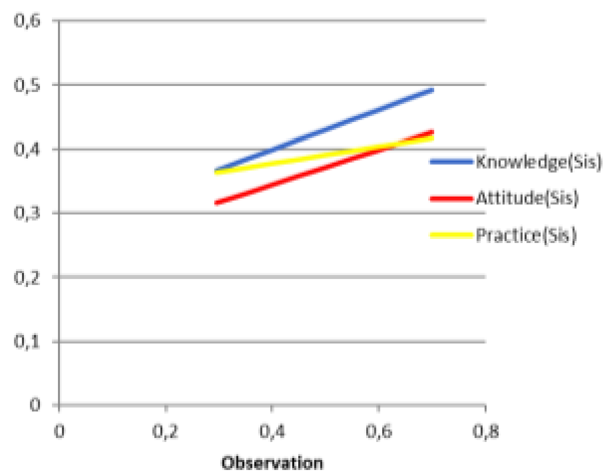

Figure 4. Development of Students' KAP due to the change observation of head master

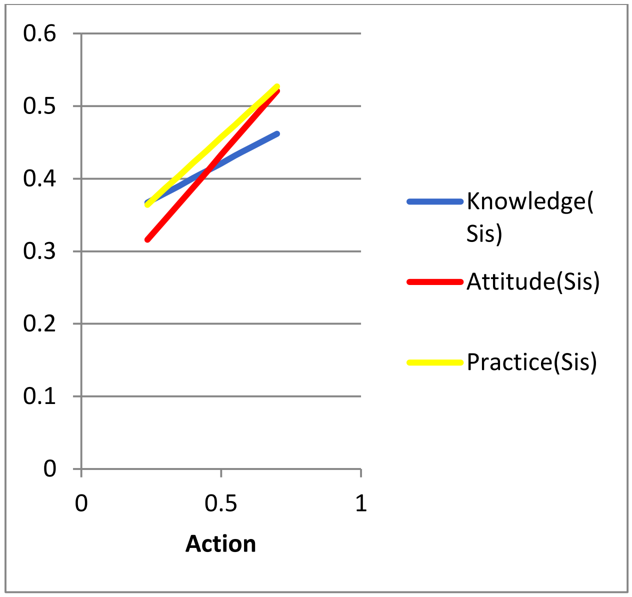

Figure 5. Development of students' KAP due to the change of head master's action

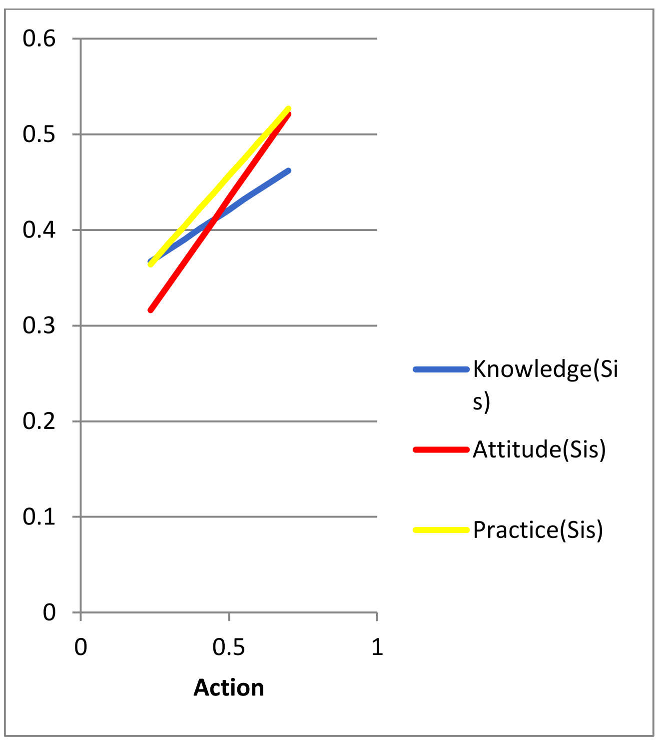

Figure 6. Development of students' KAP due to the change of Head Master's Action

Once this POMDP is completed, it is expected to produce a guide or technique that maps historical observations or behavioral steps toward action. If POMDP is used as a single agent, the school community will require a multi-agent system that reduces the number of individual POMDPs (Interactive POMDP). This method provides a decision-making sequence structure that can be used in a multi-agent environment. This model considers all information that could influence the behavior of the agents in question, such as their preferences, abilities, and beliefs. The concept of "preference" in the context of this investigation refers to what is assumed to be an internal drive as well as an outward drive and incentive. Meanwhile, knowledge and talents are assumed to be synonymous, and beliefs are assumed to be synonymous with perceptions, attitudes, and expectations.

When we speak of the multi-agent state, we are referring to the process of generalizing I-POMDPs. However, in order to ensure that this model can be calculated, the set of tabulations is assumed to be limited. This is done so that the agent's expectations of success within a given time frame are maximized. POMDP can be generalized to I-POMDP, which stands for POMDP derived from multi-agent systems, specifically ***I-POMDP$_{i,l}$* = (IS$_{i,l}$, *T$_i$*, *Ω$_i$*, *O$_i$*, *R$_i$*)**.

When it comes to specific models, specifically refers to m_j and when the model was purposefully designed, specifically refers to θ_j (Gmytrasiewicz and Doshi , 2005).

Furthermore, the agent may obtain proof or facts concerning the physical condition of the world around him and/or the activities carried out by the agent j. Agent i will compute the behavior for all of the models in m_j and then update its confidence based on a number of IS$_{i,l}$ derived from previously collected data.

There are two criteria to consider when using this reward optimization to achieve the optimal level of reward. I-POMDPs are allegedly becoming more common, allowing for the creation of multi-agent environments with few constraints. This allows for planning difficulties to arise in environments with ambiguity (uncertainty), regardless of whether agents are acting cooperatively, competitively, or in conflict with one another. In some cases, the agent is aware of the current environmental conditions and can calculate the behavior for all conditions by increasing the level of confidence, as described in the following formulation.

$$\begin{aligned} b_i^t(is^t) &= Pr\left(is^t | o_i^t, a_i^{t-1}\right) \\ &= \beta \sum_{IS^{t-1}} \sum_{a_j^{t-1}} Pr\left(a_i^{t-1} | \theta_i^{t-1}\right) O_i\left(is^t, a^{t-1}, o_i^t\right) \\ &\times \sum_{a_j^t} \tau_{\theta_j^t}\left(b_j^{t-1}, a_j^{t-1}, o_j^t, b_j^t\right) O_j\left(is_j^t, a^{t-1}, o_j^t\right) \times T_i\left(is^{t-1}, a^{t-1}, is^t\right) \end{aligned}$$

Where $\theta_i = \langle b_i, A_i, \Omega_i, T_i, O_i, R_i, OC_i \rangle$, $is = \left(s, \theta_j\right)$, $is = (s, \theta_i)$ denotes the confidence element of θ_j^{t-1} and b_j^{t-1}, respectively, β denotes the normalized constant, O_j denotes the observation function in θ_j^{t-1}, and $Pr\left(a_i^{t-1} | \theta_i^{t-1}\right)$ denotes the probability, which is the Bayesian rational a_i^{t-1} for the agent denoted by the type θ_j^{t-1}.

$\tau_{\theta_j^t}\left(b_j^{t-1}, a_j^{t-1}, o_j^t, b_j^t\right)$ is currently $Pr\left(b_i^t | b_i^{t-1}, a_i^{t-1}, o_i^t\right)$'s representative. Agent i's optimization criteria are denoted by OC_i.

POMDPs are structured similarly to POMDPs, with each confidence state in POMDP having a value that corresponds to the maximum return an agent can expect in that confidence state.

$$U(\theta_i) = \max_{a_i \in A_i} \left\{ \sum_{is} ER_i(is, a_i) b_i(is) + \gamma \sum_{a_i \in \Omega_i} Pr(o_i | a_i, b_i)\, U\left(\langle SE_{\theta_i}(b_i, a_i, o_i), \hat{\theta}_i \rangle\right) \right\}$$

Where $ER_i(is, a_i) = \Sigma_{a_j}$

$R_i\left(is, a_i, a_j\right) Pr\left(a_j | m_j\right)$ as long as $is = (s, \theta_j)$

The optimal agent action i is an action that is a component of the optimal set of actions for the $OPT(\theta_i)$ confidence state, which is defined as follows:

$$OPT(\theta_i) = \arg\max_{a_i \in A_i} \left\{ \sum_{is} ER_i(is, a_i) b_i(is) + \gamma \sum_{a_i \in \Omega_i} Pr(o_i | a_i, b_i)\, U\left(\langle SE_{\theta_i}(b_i, a_i, o_i), \hat{\theta}_i \rangle\right) \right\}$$

CONCLUSION

This study introduces a comprehensive framework for optimal sequential decision-making, tailored for governing the autonomy of agents in dynamic interactions within uncertain environments. The proposed framework is predicated on the normative planning paradigm, utilizing Markov decision processes as a foundational model, particularly focusing on Partially Observable Markov Decision Processes (POMDP). We enhance the POMDP framework to incorporate interactions among multiple agents by allowing for the inclusion of beliefs about both the physical environment and other agents. Such beliefs encompass assessments of capabilities, perceived intentions, preferences, and anticipated behaviors.

Our extended framework parallels the traditional POMDP in several respects, maintaining its core properties and similarly structured solutions, yet it uniquely accommodates the complexities of multi-agent scenarios. In the absence of other agents, our model simplifies back to the conventional POMDP.

The resultant model, termed the Coordination Mechanism-Based Model for Environmental Behavior Management (CMBEBM), is evaluated through an empirical study focusing on the development of Knowledge, Attitudes, and Practices (KAP) among students, driven by interactions defined by parameters S, Ω, and A. The findings indicate significant enhancements in students' knowledge, attitudes, and particularly actions, confirming the model's efficacy in fostering positive environmental behaviors. The CMBEBM has proven to be an effective alternative for managing environmental behaviors within school communities, providing targeted information and recommendations for strategic improvements in management practices, tailored to specific needs.

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REFERENCES

Black, P. and Wiliam, D. (1998). Assessment and classroom learning. Assessment in Education: Principles, policy & practice, 5(1), 7–74. https://doi.org/10.1080/0969595980050102

Black, P. and Wiliam, D. (2010). Inside the black box: Raising standards through classroom assessment. Phi Delta Kappan, 92(1), 81–90. https://doi.org/10.1177/0031721710092001

Boutilier, C., Dean, T., and Hanks, S. (1999). Decision-theoretic planning: Structural assumptions and computational leverage. Journal of Artificial Intelligence Research, Vol. 11, 1–94. https://doi.org/10.1613/jair.575

Duys, D. K., and Headrick, T. C. (2004). Using Markov chain analyses in counselor education research. Counselor Education and Supervision, 44(2), 108–120. https://doi.org/10.1002/j.1556-6978.2004.tb01864.x

Eid, M. and Langeheine, R. (2007). Detecting population heterogeneity in stability and change of subjective well-being by mixture distribution models. In A. D. Ong & M. H. M. van Dulmen (Eds.), Handbook of methods in positive psychology, (pp. 501–607). Oxford, NY: Oxford University Press.

Fu, W. T., and Anderson, J. R. (2006). From recurrent choice to skill learning: a reinforcement-learning model. Journal of experimental psychology: General, 135(2), p. 184. DOI: 10.1037/0096-3445.135.2.184

Gmytrasiewicz, P. J. and Doshi, P. (2005). A framework for sequential planning in multi-agent settings. Journal of Artificial Intelligence Research, Vol. 24, 49–79. DOI: https://doi.org/10.1613/jair.1579

Hauskrecht, M. (2000). Value-function approximations for partially observable Markov decision processes. Journal of Artificial Intelligence Research, Vol. 13, 33–94. DOI:10.1613/jair.678

Jennings, N. R. (1996) Coordination Techniques for Distributed Artificial Intelligence. In, O'Hare, G. M. P. and Jennings, N. R. (eds.) Foundations of Distributed Artificial Intelligence. Foundations of Distributed Artificial Intelligence (01/01/96) Wiley, pp. 187-210.

Kaelbling, L. P., Littman, M. L. and Cassandra, A. R. (1998). Planning and acting in partially observable stochastic domains. Artificial Intelligence, 101(1–2), 99–134. https://doi.org/10.1016/S0004-3702(98)00023-X

Langeheine, R., Stern, E., and Van de Pol, F. (1994). State mastery learning: dynamic models for longitudinal data. Applied Psychological Measurement, 18(3), 277–291. https://doi.org/10.1177/014662169401800308

Li, T., Choi, M., Fu, K., and Lin, L. (2019). Music sequence prediction with mixture hidden markov models. 2019 IEEE International Conference on Big Data (Big Data), IEEE, 6128–6132. DOI:10.1109/BigData47090.2019.9005695

Malone, T. W. and Crowston, K. (1994). The interdisciplinary study of coordination. ACM Computing Surveys (CSUR), 26(1), 87–119. https://doi.org/10.1145/174666.174668

Miller, G. A. (1952). Finite Markov processes in psychology. Psychometrika, 17(2), pp. 149–167. DOI: 10.1007/BF02288779

Mislevy, R. J., Steinberg, L. S., and Almond, R. G. (2003). Focus article: On the structure of educational assessments. Measurement: Interdisciplinary Research and Perspectives, 1(1), 3–62. https://doi.org/10.1207/S15366359MEA0101_02

Monahan, G. E. (1982). State of the art—A survey of partially observable Markov decision processes: theory, models, and algorithms. Management Science, 28(1), 1–16. https://doi.org/10.1287/mnsc.28.1.1

Niv, Y. (2009). Reinforcement learning in the brain. Journal of Mathematical Psychology, 53(3), 139–154.

Pata, K., Lehtinen, E., and Sarapuu, T. (2006). Inter-relations of tutor's and peers' scaffolding and decision-making discourse acts. Instructional Science, 34(4), 313–341. https://doi.org/10.1007/s11251-005-3406-5

Rony, Z. T., Wijaya, I. M. S., Nababan, D., Julyanthry, J., Silalahi, M., Ganiem, L. M., Judijanto, L., Herman, H., and Saputra, N. (2024). Analyzing the Impact of Human Resources Competence and Work Motivation on Employee Performance: A Statistical Perspective. Journal of Statistics Applications & Probability, 13(2), 787-793. DOI: 10.18576/jsap/130216

Steiner, T. E. and Greeno, J. G. (1969). An analysis of some conditions for representingN state Markov processes as general all or none models. Psychometrika, 34(4), 461–487. https://doi.org/10.1007/BF02290602

Visser, I., Raijmakers, M. E.J., & van der Maas, H. L.J. (2009). Hidden Markov models for individual time series. In J. Valsiner, P. C. M. Molenaar, M. C. D. P. Lyra, & N. Chaudhary (Eds.), Dynamic process methodology in the social and developmental sciences (pp. 269–289). Springer Science + Business Media. https://doi.org/10.1007/978-0-387-95922-1_13.

Vygotsky, L. S., and Cole, M. (1978). Mind in society: Development of higher psychological processes. Harvard University Press. https://doi.org/10.2307/j.ctvjf9vz4

Weingart, L. R., Prietula, M. J., Hyder, E. B. and Genovese, C. R. (1999). Knowledge and the sequential processes of negotiation: A Markov chain analysis of response-in-kind. Journal of Experimental Social Psychology, 35(4), 366–393. https://doi.org/10.1006/jesp.1999.1378

Wooldridge, M. (1999). Intelligent agents. Multiagent systems: A modern approach to distributed artificial intelligence, 1, 27–73.