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Tools Used from Mathematics in The Study of a Riemannian Wave Equation to Prevent the Collapse of Bridges

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Abstract

In this research, the study of dynamical systems is addressed in the area of partial differential equations, focusing on a mathematical model represented by a Dirichlet-type wave equation on a Riemannian manifold. The goal is to understand the importance of mathematics in optimizing dampers and locating them in curved structures, such as bridges, to prevent their collapse.

Keywords: *Mathematics, Dampers, Location, Bridge Collapse*

INTRODUCTION

The associated system, a Riemannian wave equation, is presented below

$$\begin{cases} u_{tt} - \Delta_g u + a(x)u_t + f(u) = \beta\, h(x) & , \text{ in } M \times \mathbb{R}^+ \\ u = 0 & , \text{ in } \partial M \times \mathbb{R}^+ \\ u(x,0) = u_0(x);\; u_t(x,0) = u_1(x) & , \; x \in M \end{cases} \tag{1}$$

which models vibrations in a curved body with Riemannian metric. The connection between physical and mathematical experience is established through Newton's Second Law, generalized in the system. The central question is raised about the possibility of demonstrating that mathematics can be used to locate and optimize the measure of dampers, avoiding the collapse of curved structures.

The proposed methodology includes the use of semigroup theory, quasi-stable systems and residual sets to demonstrate the existence, good placement and continuity of global attractors. The need for solid foundations in advanced mathematics is highlighted, such as functional analysis, partial differential equations and differential geometry in Riemannian manifolds.

In the description of the problematic reality, the problem associated with the design of curved structures, such as bridges, is mentioned, where the incorrect placement of pillars and suboptimal distances between columns can lead to collapse. The importance of mathematicians and physicists in providing theoretical and experimental support to optimize the location and measure of the dampers is highlighted, avoiding problems such as the "FLATENNING" that occurred on the Takoma and Narrows bridge in 1943 (see fig. 1) originated by aeroelasticity caused by structural forces. The research work seeks to determine if mathematical theory can optimally contribute to the placement and location of dampers in curved structures. The problem formulation

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focuses on how mathematics influences the modeling of Riemannian wave equations to prevent the collapse of these structures.

Specific problems focus on how the math optimizes the size and placement of dampers, as well as how it uses global attractors to prevent collapse. The general and specific objectives seek to demonstrate the influence of mathematics in the modeling of Riemannian wave equations and their application to prevent collapses in curved structures.

The justification stands out the importance of Riemannian Geometry on curved surfaces and stand out the innovation of the work in this context.

Good placement of the system using semigroup theory is presented as key to demonstrating the existence of global attractors and optimizing the measurement of dampers, which is essential to dissipate energy and prevent collapses in curved structures. The need for knowledge in functional spaces, Riemannian Geometry, Differential Equations and Functional Analysis is highlighted.

THEORETICAL FRAMEWORK

Background

In Martínez's thesis (Martínez, 2015), dynamic instability in Bowstring arch bridges is addressed, highlighting the parametric resonance in aesthetically attractive and slender structures, especially when faced with dynamic loads from high-speed trains. The Mathieu Equation and numerical analyzes are used to study the stability, using a bowstring model generated with SOFiSTiK.

(Couso, 2014) presents a research project on modal analysis of a square plate, explaining the differences between theoretical and experimental modal analysis. The process of theoretical modal analysis is detailed, which involves solving eigenvalue and eigenvector problems, while experimental modal analysis is based on trials with known excitation strength.

(Noel, 2023) focuses on the existence of an exponential attractor for a p-Kirchhoff model with infinite memory, studying the long-term dynamics and the interaction of the memory term with p-Laplacian and biharmonic operators in a bounded domain.

(Seminario, 2019) investigates global attractors for a viscoelastic equation with memory and nonlinear density, studying wave equations of the form

$$|\partial_t u|^p \partial_{tt} u - \Delta \partial_{tt} u - \alpha \Delta u + \int_0^\infty \mu(s)\, \Delta u(t-s) ds + f(u) = h,$$

in a bounded domain of R^3 with parameters α and p. It demonstrates the existence of a global attractor characterized by the unstable varieties of the set of stationary points of the equation.

(Bocanegra, 2019) analyzes the asymptotic dynamics of an elastic wave equation in an isotropic environment

$$\partial_t^2 u - \mu \Delta u - (\alpha + \mu) \nabla(\nabla \cdot u) + \alpha \partial_t u + u = b(x),$$

studying wave equations in a bounded domain of R^3 with Lame's parameters. It's used the classical theory of linear semigroups to demonstrate the existence of a global attractor characterized by the unstable varieties of the set of stationary points of the equation.

With respect to Differential Geometry, we see in (Cavalcanti, Ma, Marín-Rubio, & Seminario-Huertas, 2021), (Lasiecka, Triggiani, & Zhang, 2000) and (Rauch & Taylor, Exponential decay of solutions to hyperbolic equations in bounded domains, 1975), the preliminary notions with which Mathematics is supported in this research. These important topics are:

Geometry for the Optimal Region in Measure.

Optimal control condition in measure.

It is introduced the notion of a set ε-controllable in measure on a Riemannian manifold M. A set ω is considered ε-controllable if its measure and the measure of its edge in M are both less than ε. Properties of ε-controllable sets are presented, such as lock under the union, intersection, and containment in larger sets.

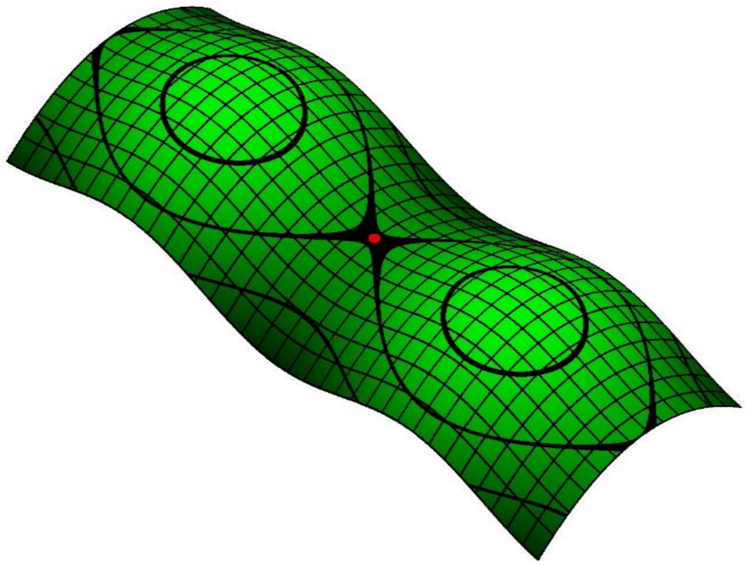

Figure 1. *Riemannian manifold*

In the theorem seen in (Yao, 2011), conditions and properties are established for the existence of open sets and functions on a Riemannian manifold that satisfy certain properties. These constructions are used to characterize admissible ε-controllable regions. Criteria are established for the choice of ε_0 in the theorem and the key properties of the constructions carried out as well as the composition into overlapping sets are highlighted. The concept of a family of overlapping sub-domains on the manifold M is introduced. Then, it is presented a theorem that proves the existence of overlapping subdomains that satisfy certain properties, related to the function d. It is stated that, given certain conditions on the function d and a ε-controllable region ω, there exists a collection of overlapping sub-domains that satisfy specific properties.

In summary, it develops a geometric and analytical framework to deal with problems related to optimal control in measure on a Riemannian manifold. ε-controllable sets and overlapping sub-domains are used to set conditions on functions and regions of interest.

Partial Differential Equations (PDEs)

Partial differential equations play a central role in mathematical modeling, providing a framework for describing the behavior of systems that evolve over time and space. From classical physics to modern engineering, PDEs are used to study diverse phenomena such as heat transfer, fluid dynamics, electromagnetism, and quantum mechanics. In this report, we delve into the theory of PDEs, examining their mathematical properties, solution techniques, and applications in science and engineering. A more extensive discussion of his theory can be found in (Evans, 2010), (Strauss, 2007) and (Farlow, 1993).

At their core, partial differential equations are equations that involve partial derivatives of an unknown function with respect to multiple independent variables. Depending on their form and properties, PDEs can be classified into various types, including elliptic, parabolic, and hyperbolic equations. The theory of PDEs encompasses a wide range of mathematical concepts and techniques, including existence and uniqueness theorems, classification schemes, and qualitative properties of solutions.

Solving PDEs analytically is often challenging, and researchers rely on a variety of numerical and computational techniques to approximate solutions. Finite difference, finite element, and spectral methods are among the most commonly used approaches for solving PDEs numerically. These methods discretize the domain of the problem and approximate the derivatives using finite or spectral expansions, leading to systems of algebraic equations that can be solved using iterative or direct methods.

Partial differential equations find numerous applications across science and engineering, driving advancements in fields such as physics, biology, chemistry, and materials science. In physics, PDEs are used to model the behavior of physical systems, including fluid flow, wave propagation, and quantum mechanics. In engineering, PDEs are employed to design and analyze complex systems such as heat exchangers, aircraft wings, and electronic circuits. Moreover, PDEs play a crucial role in mathematical biology, where they are used to model biological processes such as population dynamics, biochemical reactions, and neural activity.

Despite the advancements in numerical methods and computing technology, solving PDEs remains a challenging task, particularly for high-dimensional problems and nonlinear equations. Future research directions in the field of PDEs include the development of efficient and scalable algorithms, the integration of machine learning techniques, and the exploration of new mathematical formulations and solution strategies. Additionally, interdisciplinary collaborations between mathematicians, scientists, and engineers are essential for tackling complex PDE problems and addressing real-world challenges.

Dynamic Systems

Dynamic systems represent a broad class of mathematical models that describe how quantities change over time in response to internal and external influences. From classical mechanics to modern control theory, dynamic systems provide a unified framework for analyzing the behavior of physical, biological, and socio-economic systems. In this report, we embark on a journey to explore the intricacies of dynamic systems, unraveling their mathematical foundations, conceptual frameworks, and practical applications in science and engineering. The reader interested in deepening this theory can take into account the following references: (Meiss, 2007) and (Hirsch & Devaney, 2012).

At the heart of dynamic systems theory lies the concept of state variables, which represent the evolving internal states of a system. These variables are typically governed by differential or difference equations that capture the system's dynamics over time. The behavior of dynamic systems is characterized by trajectories in state space, which represent the evolution of the system's state variables over time. Stability analysis plays a crucial role in assessing the long-term behavior of dynamic systems, identifying equilibrium points, limit cycles, and other attractors that govern the system's dynamics.

Dynamic systems can be described using various mathematical formalisms, including ordinary differential equations (ODEs), PDEs, difference equations, and stochastic differential equations (SDEs). ODEs are particularly common in modeling continuous-time systems, while discrete-time systems are often described using a difference equation framework. Stochastic differential equations extend the theory of dynamic systems to account for random fluctuations and noise in the system dynamics, providing a more realistic representation of many real-world phenomena.

Stability analysis is a fundamental aspect of dynamic systems theory, focusing on the qualitative behavior of system trajectories over time. Linear stability analysis involves analyzing the eigenvalues of the system's Jacobian matrix to determine the stability of equilibrium points. Lyapunov stability theory provides a more general framework for assessing the stability of nonlinear dynamic systems, using Lyapunov functions to quantify the system's stability properties and establish stability criteria.

Riemannian Wave Equations

Riemannian wave equations are fundamental tools in the study of differential geometry and the theory of general relativity. These equations describe how perturbations propagate in a curved spacetime, such as gravitational waves in general relativity theory. However, in the context of localization and optimization of damper measurements, we can adapt these concepts to address problems of control and optimization of damped mechanical systems.

To solve the optimization problem, we can resort to variational techniques or numerical methods such as the finite element method. These methods allow us to find the optimal distribution of dampers that minimizes the Riemannian metric, that is, maximizes energy absorption efficiency.

It is important to consider practical limitations, such as space constraints, costs, and available materials, when designing and implementing the optimized damper distribution. Experimental validation also plays a crucial role in ensuring that the proposed damper distribution meets the expected performance requirements.

For a more in-depth study of these equations and its applications, we recommend the reader the following alternatives: (Petersen, 2006), (Wald, 1984) and (Choquet-Bruhat & DeWitt-Morette, 2000). For practicality, below we highlight the main concepts around these equations:

Riemannian Geometry

The cornerstone of Riemannian geometry is the concept of a Riemannian manifold, which consists of a smooth manifold equipped with a Riemannian metric tensor. This tensor endows the manifold with a notion of distance and angle, allowing for the measurement of geometric properties such as curvature and geodesic paths. The curvature of a Riemannian manifold is encapsulated by the Riemann curvature tensor, a fundamental object that quantifies the deviation of parallel transport around infinitesimal loops. Through the machinery of differential geometry, Riemannian geometry provides a rigorous mathematical framework for analyzing curved spaces of arbitrary dimension and topology.

The historical development of Riemannian geometry is a testament to the ingenuity of mathematicians and physicists across centuries. Bernhard Riemann's seminal habilitation lecture in 1854 laid the groundwork for the subject, introducing the revolutionary concept of higher-dimensional spaces endowed with variable curvature. Subsequent luminaries such as Élie Cartan, Henri Poincaré, and David Hilbert further advanced the theory, uncovering profound connections between geometry, topology, and physics. The advent of general relativity in the early 20th century marked a watershed moment for Riemannian geometry, with Albert Einstein's field equations providing a geometric description of gravitation as the curvature of spacetime.

Beyond its foundational principles, Riemannian geometry finds a myriad of applications across diverse scientific disciplines. In theoretical physics, it serves as the mathematical backbone of general relativity, offering a geometric framework for understanding the dynamics of spacetime and gravitational interactions. In differential topology, it underpins the study of manifolds, providing tools for classifying and analyzing their geometric properties. In applied mathematics, Riemannian geometry finds applications in fields such as machine learning, optimization, and computer graphics, offering powerful techniques for analyzing and processing data on non-Euclidean spaces.

In the contemporary scientific landscape, Riemannian geometry continues to play a central role in shaping our understanding of the natural world and driving technological innovation. From cosmology to robotics, from medical imaging to artificial intelligence, the principles of Riemannian geometry permeate a wide array of disciplines, providing insights into the structure and behavior of complex systems. As we delve deeper into the mysteries of the universe and confront the challenges of the digital age, the enduring relevance of Riemannian geometry underscores its status as a cornerstone of modern mathematics and physics.

Propagation of Perturbations

Riemannian wave equations govern the propagation of perturbations, such as gravitational waves or other types of waves, in Riemannian manifolds. They determine how these perturbations evolve over time and space, influenced by the curvature of the underlying geometry.

From a physical perspective, Riemannian wave equations offer insights into the behavior of waves in curved spaces and their interaction with gravitational fields. Gravitational waves, for instance, are ripples in the fabric of spacetime that propagate outward from accelerating masses, carrying energy and information across the universe. Riemannian wave equations provide a rigorous mathematical framework for studying the generation, propagation, and detection of gravitational waves, offering predictions that have been confirmed by observations.

Mathematical Formulation

Mathematically, Riemannian wave equations can be expressed as partial differential equations involving the metric tensor, the Laplace-Beltrami operator, and the wave operator. Solutions to these equations provide insights into the behavior of waves in curved spaces and can be used to make predictions about phenomena such as gravitational wave propagation.

Understanding these basic concepts provides a foundation for exploring the implications of Riemannian wave equations in various fields, including theoretical physics, cosmology, and mathematical physics.

Global Attractors

Global attractors represent the ultimate fate of dynamical systems, capturing the long-term behavior towards which trajectories converge over time. These attractors arise in diverse contexts, from physical systems governed by differential equations to biological networks and economic models. In this report, we embark on a journey to explore the theory of global attractors, examining their mathematical underpinnings, geometric properties, and broader implications for understanding the dynamics of complex systems. For more information regarding the theory of global attractors we recommend the following references: (Temam, 1997), (Robinson, Infinite-Dimensional Dynamical Systems: An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors, 2001) and (Robinson, Attractors and Bifurcations: Analysis and Numerical Procedures, 1995).

Mathematical Foundations

At its core, the theory of global attractors is grounded in the mathematics of dynamical systems, which encompasses a wide range of mathematical techniques and tools for analyzing the behavior of evolving systems over time. Global attractors arise as invariant sets that attract the trajectories of dynamical systems, providing a stable endpoint towards which nearby trajectories converge. Mathematically, global attractors are often characterized as compact, invariant, and minimal sets within the phase space of a dynamical system, encapsulating the essential dynamics of the system.

Geometric Interpretations

From a geometric perspective, global attractors offer insights into the underlying structure and organization of complex systems. These attractors can take on various forms, ranging from simple fixed points and limit cycles to more intricate structures such as strange attractors and chaotic sets. The geometry of global attractors reflects the underlying dynamics of the system, revealing patterns of stability, periodicity, and chaos that emerge from the interplay of nonlinear interactions and feedback mechanisms. By visualizing global attractors in phase space, researchers can gain a deeper understanding of the behavior of dynamical systems and uncover hidden patterns and structures.

Practical Implications

Global attractors have practical implications across a wide range of scientific disciplines, influencing our understanding of phenomena as diverse as climate dynamics, neural networks, and economic systems. In climate science, for example, global attractors play a key role in modeling the long-term behavior of the Earth's climate system, helping researchers predict future trends and assess the impact of environmental changes. In neuroscience, global attractors provide a framework for understanding the dynamics of neural networks and the emergence of complex patterns of activity underlying cognitive processes. Similarly, in economics, global attractors shed light on the stability and resilience of economic systems, informing policy decisions and risk management strategies.

HYPOTHESES AND VARIABLES

The hypotheses are formulated around the general problem, highlighting the influence of mathematics in the modeling of a Riemannian wave equation to prevent the collapse of curved structures in Civil Engineering. The specific hypotheses focus on the optimization of the measure and location of dampers, as well as the use of global attractors to avoid collapses.

Regarding conceptual variables, "Mathematics" is presented as an exact science that uses partial differential equations and dynamical systems to support Civil Engineering in the theory of continuity of global attractors and the optimization of dampers. "Modeling a Riemannian Wave Equation" involves the mathematical representation of displacements, velocities and accelerations in curved structures.

In the system (1) the mathematical entities are described, these are:

u: Displacements of wave vibrations.

a(x)u_t: Displacements of wave vibrations.

a(x): Location.

f(u): Nonlinear structural force.

h(x): External force.

MATERIALS AND METHODS

The methodological design of the research is classified as basic, using an inductive-deductive approach. Graphically, the design is represented with two main variables: Mathematics (V1) and Modeling of a Riemannian wave equation (V2). The research method applied is desktop or library, focused on an exhaustive analysis of the bibliography related to the topic. Given the abstract nature of the work, there is no defined population or sample. The study location includes relevant spaces such as the Faculty of Industrial and Systems Engineering of the National University of Callao, as well as the UNAC Central Library. Data collection is carried out mainly through the review of specialized literature, and statistical procedures are not required due to the abstract nature of the work.

MATHEMATICAL TOOLS USED

Damper: Capacity of a system to dissipate kinetic energy into other forms of energy.

Global Attractor: Set of values towards which a system tends to evolve, regardless of the initial conditions.

Bridge Collapse: It can be caused by various factors such as loss of coating on the reinforcement, incorrect waterproofing, concreting under extreme conditions, among others.

Continuity: Property of a function for which small variations in the domain result in small variations in the values of the function; fundamental in mathematical analysis and topology.

Wave Equation: Mathematical description of interacting waves, analyzing the behavior when dividing the wave into components.

Partial Differential Equation (PDE): Involves partial derivatives of an unknown function with two or more independent variables.

Differential Geometry: Studies smooth manifolds with additional geometric structures, such as Riemannian metrics.

Riemannian Geometry: Study of differential manifolds with Riemann metrics that assign quadratic forms to tangent spaces.

Mathematics: Science of structure, order and repetitive factors, based on counting, measuring and describing shapes.

Measure (in Mathematics): Generalized concept of geometric measurements and probability in measurement theory.

Modeling a Partial Differential Equation (PDE): Mathematical representation of a PDE, such as the basic SIR model in epidemiology.

Dynamic Systems: Study of evolution equations to understand long-term behavior, using concepts of topology and analysis.

Riemannian manifold: In Riemannian geometry, a differentiable manifold equipped with an inner product on each tangent space that varies smoothly point to point.

One application is the Important Theorem seen in (Mendoza-Arenas, y otros, 2023), which proves the existence of a global Attractors for a Riemannian wave equation and the optimal measure of localized damping.

The mathematical results seen in (Cavalcanti, Ma, Marín-Rubio, & Seminario-Huertas, 2021), (Lasiecka, Triggiani, & Zhang, 2000), (Ma & Seminario-Huertas, 2020), (Rauch & Taylor, Decay of solutions to nondissipative hyperbolic systems on compact manifolds, 1975), (Triggiani & Yao, 2002), and (Yao, 2011) are reflected in (Mendoza-Arenas, y otros, 2023), which show the existence of a global attractor of an equation of Riemannian wave with localized damping. Here we mention the important points:

General objective: Study the system (2), which is a semilinear wave equation with damping dissipation, denoted by β called the perturbation coefficient.

$$\begin{cases} \partial_t^2 u - \Delta_g u + a(x)\partial_t u + f(u) = \beta h(x) & , \text{ in } M \times (0,\infty) \\ u = 0 & , \quad on\ \partial M \times (0,\infty) \\ u(x,0) = u_0(x),\ \partial_t u(x,0) = u_1(x) & , \quad x \in M. \end{cases} \tag{2}$$

Theoretical Methodology: The need for a unique continuation property and observability inequalities is mentioned. The proposed approach differs from previous methods that required high regularity in the solutions. Instead, it is based on concepts developed in works such as (Cavalcanti, Ma, Marín-Rubio, & Seminario-Huertas, 2021), (Triggiani & Yao, 2002) and (Lasiecka, Triggiani, & Zhang, 2000).

Concept of Spatial Vector Fields: It is used

the concept of spatial vector fields, which involves dividing the edge ∂M with respect to the sign of H, where H is a strategic vector field. This approach requires less regularity and follows Carleman's estimates.

Linear Wave Equation: A linear wave equation is presented on the compact Riemannian manifold (M,g) with an additional term χω∂tu, where Δ is the Laplace-Beltrami operator, and χω is the characteristic function of an open subset ω of M. The initial and boundary conditions are specified.

System Energy: The system energy is defined in terms of the time velocity ∂tu and the gradient (∇u), using the Levi-Civita connection on the manifold.

Geometric Control Condition (GCC): It is mentioned that the energy of the system decays

exponentially to zero if and only if ω satisfies the geometric control condition (GCC). This condition implies that generalized geodesics traveling with speed 1 enter ω before a specific time t_0.

Control of the Measure of the Observation Region ω: Discussed the importance of controlling the measurement of the observation region ω. It is introduced the concept of ε-controllability (in measure) and highlights the question of whether the edge measure can be arbitrarily small for sets that comply with GCC.

Focus on Semilinear Equations: It is indicated that the main objective is to study the long-term dynamics of semilinear equations with effective damping dissipation in a region ε-controllable.

Expected Results: The study aims to demonstrate that the system (2) is well placed and it has a regular global attractor with finite fractal dimension Aβ. Furthermore, we seek to show the convergence of the stabilization regions when β varies in the interval [0,1].

An advanced mathematical study is described that combines concepts from Riemannian geometry, linear wave equations, geometric control, and results from previous work to address the dynamics of semilinear equations in ε-controllable regions.

Important References: Reference is made to several previous works, such as (Bardos, Lebeau, & Rauch, 1992), (Lasiecka, Triggiani, & Zhang, 2000), (Miller, 2003), (Yao, 2011), among others. These works provide the theoretical and methodological bases for the proposed study.

In summary, the work is focused on developing theoretical results on the dynamics of semilinear waves in the presence of damping dissipation in a ε-controllable region, using novel approaches based on spatial vector fields and Carleman concepts.

RESULTS

Global attractors are regions of compact stabilization.

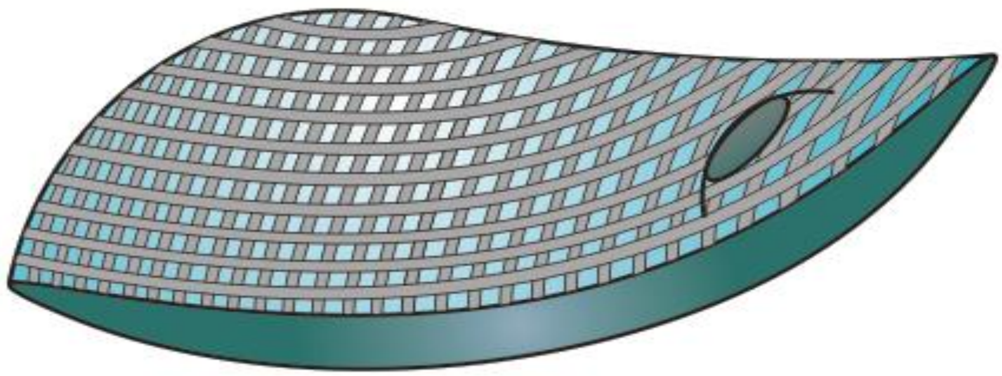

Figure 2. *The mesh on the variety M represents the region ω*

Figure 4. Tbilisi Bridge in Georgia

Figure 5. Dome Houses

Figure 6. Chilina Bridge in Arequipa

Figure 7. Well located shock absorber

In figure 3, the region ω is an open of M, it represents the dissipation or damping region

In figure 4, the curved structure, similar to a hyperbolic paraboloid, is designed that way, since the tube mesh is the compact stabilization region that prevents the bridge from collapsing.

In figure 5, in Dome houses, which have a hemispherical shape, the tube mesh serves as a damper.

In figure 6, the bridge has a solid structure that unlike Takoma and Narrows, the construction consists of strong metal structures and the pylons serve as dampers.

Finally, we observe in Figure 7 that the bridge has an optimally placed damper and is well located.

CONCLUSIONS

It is concluded first of all that Mathematics has topics and tools such as the study of a Riemannian wave equation that models physical phenomena.

Another important point is that if the curved structures, such as bridges, have Riemannian curvature, the dampers to be placed will serve as support and will avoid the collapse of said structures, due to physical phenomena.

Over time, structures improve due to a mathematical theory such as Partial Differential Equations, which supported by Functional Analysis and Riemannian Geometry contribute to the protection of human life.

The good location of dampers and the optimization of the measurement is also an important factor for the design of a bridge or a curved structure.

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