

# Measuring A Composite Financial Inclusion Index in Lower-Middle Income Countries

Nguyen Cao Anh<sup>1</sup> and Thai Hong Thuy Khanh<sup>2\*</sup>

## Abstract

*This study suggests a composite index of financial inclusion (FI) for financial development, based on three key dimensions: penetration, availability, and usage in lower-middle income countries. The methodology uses principal component analysis PCA to exploit a composite index of financial inclusion and test its impact of new components on financial development. On the secondary data of financial access from the sources of FAS-IMF and WB-Index, the research result is found that there are two new components of overall financial inclusion: the traditional financial inclusive index (TFI) includes three traditional dimensions: traditional penetration (TP), traditional availability (TA), and traditional usage (TU); and the digital financial inclusive index (DFI) includes three digital dimensions: digital penetration (DP), digital availability (DA), and digital usage (DU). The research findings give the empirical evidence that the digitalized level of financial inclusion is quite weak in lower-middle income countries. The research contributes to the methodology of building a composite index of financial inclusion with optimal weights of dimensions and traditional-digital financial inclusive indexes.*

**Keywords:** Composite FI Index, Financial Access, Financial Inclusion, FinTech, Principal Component Analysis

## INTRODUCTION

Financial inclusion is a keyword for exploit much in recent studies related to individual or institutional financial access with usefulness and affordability in providing financial products and services from traditional facilities to digital facilities, such as transactions, payments, savings, credit and insurance that meet their needs. These financial inclusive activities contribute to economic growth [5; 14], and how to build a composite index of financial inclusion for supporting financial system in lower-middle income countries is the main heart of this research.

In some recent reviews of financial inclusion, financial access facilities have been developed by the FinTech, defined as new technology that seeks to improve and automate the delivery and use of financial services as well as the provision of financial products, such as the comparison of traditional and digital financial inclusion [6; 12; 17; 24]; the impact of financial inclusion on economic growth [13; 22; 23] to unlock new business opportunities for the individuals and companies when they had easy financial access, especially the development of digital technologies in the users' mobile apps. Therefore, there are two main assumptions in constructing a composite index of financial inclusion:

### Assumption 1: Evaluating Digitized Level of Financial Inclusion in Lower-Middle Income Countries.

Beyond the pressure of competitiveness, commercial banks and securities use digital technologies to enhance the usage of digital financial products and services [24]. The construction of digital financial inclusion includes three dimensions: coverage breadth, usage depth, and digitization level. The first dimension describes the number of people using electronic accounts to reflect the financial environment that people could approach; the second dimension reveals the regional-level usage of digital financial inclusion to reflect the provision ability of financial products and services; and the third dimension evaluates efficiency of digital financial inclusion with a reasonable cost. Though this approach is quite attractive in financial innovation, investment cost of

---

<sup>1</sup> Faculty of Finance and Accounting, Nguyen Tat Thanh University, HoChiMinh City, Viet Nam, E-mail: [ncanh@ntt.edu.vn](mailto:ncanh@ntt.edu.vn)

<sup>2</sup> Faculty of Finance and Accounting, Nguyen Tat Thanh University, HoChiMinh City, Viet Nam, E-mail: [thtkhanh@ntt.edu.vn](mailto:thtkhanh@ntt.edu.vn) (Corresponding Author)

digital financial facilities is quite high because the digital financial access has still been barriers of financial efficiency in ASEAN countries. Therefore, seeking a composite index of financial inclusion is based on a combination of the traditional financial inclusion (TFI) and the digital financial inclusion (DFI).

Assumption 2: Determining optimal weights of financial inclusion.

The research of Banik & Roy [6] provides weights of the traditional financial inclusion (TFI) and the digital financial inclusion (DFI) for the overall financial inclusion (FI) by the method of a three-stage principal component analysis PCA. However, a PCA equation of component with 02 dimensions TFI and DFI always finds out the equal loading scores in all cases. In fact, the weights of financial inclusion are different between TFI and DFI because the transformative ability from TFI to DFI has increased in lower-middle income countries. Therefore, this research suggests an alternative measure of optimal weights between TFI and DFI by a Sharpe ratio to form an allocation line of financial inclusion (FIAL).

The remainders of this research include the contents as follows: literature reviews of financial inclusion, a proposed model, research methodology, research results and discussions, and conclusions.

## II. Literature Reviews

Measuring overall financial inclusion (FI) relies on the conditions of social economics in countries which their financial access has some barriers of financial-system development. In the scope of this research, we gather some definitions, determinants, research methodology to measure a composite index of financial inclusion in theoretical framework.

### **Definition of traditional and digital financial inclusion**

In the most basic definition, financial inclusion refers to the fact that a person owns such an account at a formal financial institution that allows to save and borrow money, to contract insurance or to use payment services in the traditional way [28].

According to AFI [2], financial inclusion refers to a state that all working age adults have effective access to credit, savings, payments, and insurance from formal service providers, and effective access involves convenient and responsible service delivery, at a cost affordable to the customer and sustainable for the provider, with the result that financially excluded customers use formal financial services rather than existing informal options.

On the development of financial technology, financial inclusion is divided into two groups: traditional financial inclusion (TFI) and digital financial inclusion (DFI). Therefore, Ozili [19] gives a definition of digital financial inclusion that involves bringing unbanked adults into the formal financial sector by offering financial services to unbanked adults using devices that have a digital interface such as a mobile phone or other digital devices.

In the lower-middle income countries, evaluating digitized level of financial inclusion has been limited on the financial infrastructure, and the combination of traditional and digital financial inclusion is a priority condition to construct a composite index of financial inclusion.

### **Determinants of Financial Inclusion**

Many previous studies have attempted to seek the determinants of financial inclusion by using the methods of principal component analysis (PCA) to exploit a composite index of financial inclusion, and the most focus on the following dimensions of financial inclusion [21; 24].

### **Dimensions of Financial Inclusion**

Dimensions of financial inclusion are developed by the study of Sarma [25], based on three key dimensions: penetration, availability, and usage. On the modification of financial inclusion, these dimensions are found on the principal component analysis PCA for a composite index of financial inclusion [4; 6; 7; 13; 16; 18; 24; 27].

### **Financial Penetration (FP)**

Financial penetration (FP) is a dimension of financial inclusion to reflect the coverage breadth or the depth of outreach that people could access to financial products and services in the whole social economics. The keywords of the coverage breadth and the outreach depth represent the financial access that people in a region or a country could catch up convenient investment/business opportunities in financial transactions.

**Financial Availability (FA)**

Financial availability (FA) is a dimension of financial inclusion to reflect the conditions of financial resources or infrastructure that financial institutions are willing to provide financial products and services.

**Financial Usage (FU)**

Financial usage (FU) is a dimension of financial inclusion to imply how clients use financial services, such as the regularity and duration of the financial product/service over time (e.g. average savings balances, number of transactions per account, number of electronic payments made).

**Classification of Traditional and Digital Financial Inclusive Indicators**

Numerous indicators are used for the construction of financial inclusion. Table 1 shows items used by previous researchers for the description of proxy variables in designing a composite index of financial inclusion. Based on three dimensions of financial inclusion, the classification of traditional and digital financial inclusive indicators is throughout the transactions of traditional facilities and modern facilities (Khera et al., 2021). Some characteristics of digital transactions use financial technologies on mobile devices in digital payments, mobile banking, mobile money, and others, different from some characteristics of traditional transactions that use registered accounts, cards and automated teller machines ATM in financial system. The studies of Banik & Roy [6], Florian [8], and Khera et al. [13] classify some traditional and digital financial inclusive indicators as follows:

Table 1. The classification of traditional and digital financial inclusive indicators

Dimensions	Traditional FI indicators		Digital FI indicators	
	Indicators (Variables)	Sources	Indicators (Variables)	Sources
Penetration (FP)	Number of deposit accounts with commercial banks per 1,000 adults (TPDA)	IMF-FAS	Number of registered mobile money accounts per 1,000 adults (DPNA)	IMF-FAS
	Number of loan accounts with commercial banks per 1,000 adults (TPLA)	IMF-FAS	Number of registered mobile money agent outlets per 100,000 adults (DPAO)	IMF-FAS
	Number of credit cards per 1,000 adults (TPNC)	IMF-FAS	Number of registered mobile money agent outlets (DPAK)	IMF-FAS
	Number of debit cards per 1,000 adults (TPND)	IMF-FAS		
Availability (FA)	Number of commercial bank branches per 1,000 km <sup>2</sup> (TANB)	IMF-FAS	Mobile money agent outlets: active per 100,000 adults (DAAO)	IMF-FAS
	Number of ATMs per 1,000 km <sup>2</sup> (TANK)	IMF-FAS	Number of active mobile money accounts per 1,000 adults (DAAA)	IMF-FAS
Usage (FU)	Outstanding deposits with commercial banks on percent of GDP (TUOD)	IMF-FAS	Value of mobile money transactions on percent of GDP, during the reference year (DUVM)	IMF-FAS
	Outstanding loans from commercial banks on percent of GDP (TUOL)	IMF-FAS	Number of mobile money transactions per 1,000 adults, during the reference year (DUNT)	IMF-FAS

Table 1 shows that the advances of financial technologies changed from traditional financial inclusion (TFI) to digital financial inclusion (DFI) in the processes of financial development, which financial institutions expect to exploit the utilization of three dimensions: penetration (FP), availability (FA) and usage (FU) in the measurement of overall financial inclusion for the purpose of their competitive advantages.

### Measurement of Distance-Based Financial Inclusive Index

The measurement of distance-based financial inclusive index is derived from the multi-dimensional Euclidean distance to coordinate points by using Pythagorean theorem, so-called Pythagorean distance, shown in Figure 1.

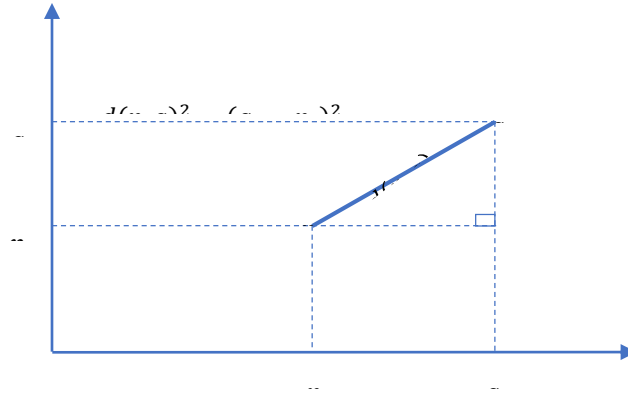


Figure 1 indicates that the measurement of Euclidean distance is based on the distance between two points  $p$  and  $q$  on each dimension estimated by Pythagorean theorem. However, the measures of financial inclusive index based on multi-dimensional Euclidean distance are various methods [21]. According to studies of Sarma [26]:

$$FI = \frac{1}{2}(X_1 + X_2) = \frac{1}{2} \left[ \sqrt{\frac{d_p^2 + d_A^2 + d_U^2}{w_p^2 + w_A^2 + w_U^2}} + \left( 1 - \sqrt{\frac{(w_p - d_p)^2 + (w_A - d_A)^2 + (w_U - d_U)^2}{w_p^2 + w_A^2 + w_U^2}} \right) \right], \quad (1)$$

in which  $FI$  is overall financial index,  $d_p$  is dimension of penetration with the weight  $w_p$ ,  $d_A$  is dimension of availability with the weight  $w_A$ , and  $d_U$  is dimension of usage with the weight  $w_U$ . The formula (1) is only based on a simple average of normalized Euclidian distance  $X_1$  and normalized inverse Euclidian distance  $X_2$  with the formula of each dimension  $i$  with the weight  $w_i$ :

$$d_i = w_i \frac{A_i - m_i}{M_i - m_i}, \quad (2)$$

where  $d_i$  is  $i^{th}$  dimension value with  $\{i = P, A, U\}$ ,  $A_i$  is actual indicator value of dimension  $i$ ,  $m_i$  is lower bound on dimension  $i$ , and  $M_i$  is upper bound on dimension  $i$ , and the weight  $w_i$  is known coefficient in which importance of penetration  $d_p$  is higher than the ones of availability  $d_A$  and usage  $d_U$ , hence the research of Sarma [26] sets three known weights of financial inclusive index  $\{w_p, w_A, w_U = 1, .5, .5\}$  under the importance of dimensions. In the research of Ren et al. [24], weights of financial inclusive index are measured by the dimensional variation:

$$w_i = \frac{CV_i}{\sum CV_i} = \frac{S_i/\bar{A}_i}{\sum S_i/\bar{A}_i}, \quad (3)$$

where  $CV_i$  is variation of dimension  $i$ ,  $S_i$  is standard deviation of dimension  $i$ , and  $\bar{A}_i$  is mean value of dimension  $i$ . Due to the disunity of the dimensions of different indicators, the variation coefficient  $CV_i$  is

employed to determine the weight of dimension  $i$ , and value of dimension  $d_i$  is coefficient with the range from 0 to 1 to reduce biasness among various indicators. In addition, categorizing the overall FI index in sample data is determined by 3 levels of FI index [9; 25].

Table 2. Categories of financial inclusive index

Author (Year)	Level	Categories
Sarma [25]	$0 \leq FI \leq 0.3$ $0.3 < FI \leq 0.5$ $0.5 < FI \leq 1$	Low financial inclusion Medium financial inclusion High financial inclusion
Goel & Sharma [9]	$0 \leq FI \leq 0.4$ $0.4 < FI \leq 0.6$ $0.6 < FI \leq 1$	Low financial inclusion Medium financial inclusion High financial inclusion

Out of the Euclidean-distance measurement of distance-based financial inclusion index in the formula (1), there is another measurement of correlation-based financial inclusion index by the principal component analysis PCA [6; 11; 15; 20; 24], finding out a composite index of financial inclusion. In the context, we use a combination of traditional financial inclusion and digital financial inclusion among three dimensions: penetration, availability and usage.

**A PROPOSED MODEL**

A proposed model is derived from three main studies of Banik & Roy [6], Khera et al. [13] to measure the overall financial inclusion (FI) based on three dimensions of penetration (FP), availability (FA) and usage (FU) in a combination of traditional financial inclusion (TFI) and digital financial inclusion (DFI) in ASEAN countries:

$$\begin{aligned}
 FI &= w_{TFI}TFI + w_{DFI}DFI \\
 &= w_{TP}TP + w_{TA}TA + w_{TU}TU + w_{DP}DP + w_{DA}DA + w_{DU}DU + \varepsilon_{FI},
 \end{aligned}
 \tag{4}$$

in which  $TP$  is dimension of traditional financial penetration with weight score  $w_{TP}$ ,  $TA$  is dimension of traditional financial availability with weight score  $w_{TA}$ ,  $TU$  is dimension of traditional financial usage with weight score  $w_{TU}$ ,  $DP$  is dimension of digital financial penetration with weight score  $w_{DP}$ ,  $DA$  is dimension of digital financial availability with weight score  $w_{DA}$ ,  $DU$  is dimension of digital financial usage with weight score  $w_{DU}$ , and  $\varepsilon_{FI}$  is the modelling error. The equation (4) is a combination of traditional financial inclusion (TFI) and digital financial inclusion (DFI) to explore a composite index of financial inclusion in lower-middle income countries by a two-stage principal component analysis (two-stage PCA).

**Research Methodology**

The construction of a composite financial inclusion index is derived from the studies related to the method of principal component analysis PCA [6; 11; 15; 20; 24], and using the eigenvalue coefficient of component  $\lambda_i$  represents the standard deviation of the component for estimating the component weight  $w_i$  on overall financial inclusion:

$$w_i = \frac{\lambda_i}{\sum_{i=1}^n \lambda_i}.
 \tag{5}$$

with  $n$  dimensions. The different point of this research is to use a Sharpe ratio, defined as a ratio between expected distance and standard deviation of 06 dimensions in the equation (4) to find out their optimal combination through the following Lagrange function  $\mathcal{L}$ :

$$\mathcal{L}(\omega, v, \mu) = \frac{1}{2}\omega^T \Sigma_d \omega + v[1 - \omega^T [1]] + \mu[E(FI) - \omega^T E(d)], \tag{6}$$

in which  $\Sigma_d$  is variance-covariance matrix of 06 dimensions,  $E(d)$  is expected distance matrix of 06 dimensions,  $E(FI)$  is expected distance for overall financial inclusion,  $[1]$  is matrix of vector 1 with total weights  $[1]^T \omega = 1$ ,  $\omega$  is weight matrix of 06 dimensions,  $v$  is Lagrange multiplier of the dimensional weights,  $\mu$  is Lagrange multiplier of the dimensional expected distances, and symbol  $[...]^T$  is transpose matrix. Finding out optimal-weights of 06 dimensions in overall financial inclusion is based on the maximized Sharpe-ratio between the expected distance  $E(FI)$  and the standard deviation  $S_{FI} = \sqrt{\omega^T \Sigma_d \omega}$  through the allocation line of dimensions for overall financial inclusion (ALD):

$$E(FI)^* = a_d + \omega_d \times [E(FI) - a_d], \tag{7}$$

where  $E(FI)^*$  is optimal expected distance of dimensions for overall financial inclusion,  $a_d$  is stable distance of dimensions for overall financial inclusion,  $S_{FI}$  is standard deviation of dimensions for overall financial inclusion, and  $\omega_d$  is the weight of dimensions for overall financial inclusion in the combination between expected distance and standard deviation of dimensions.

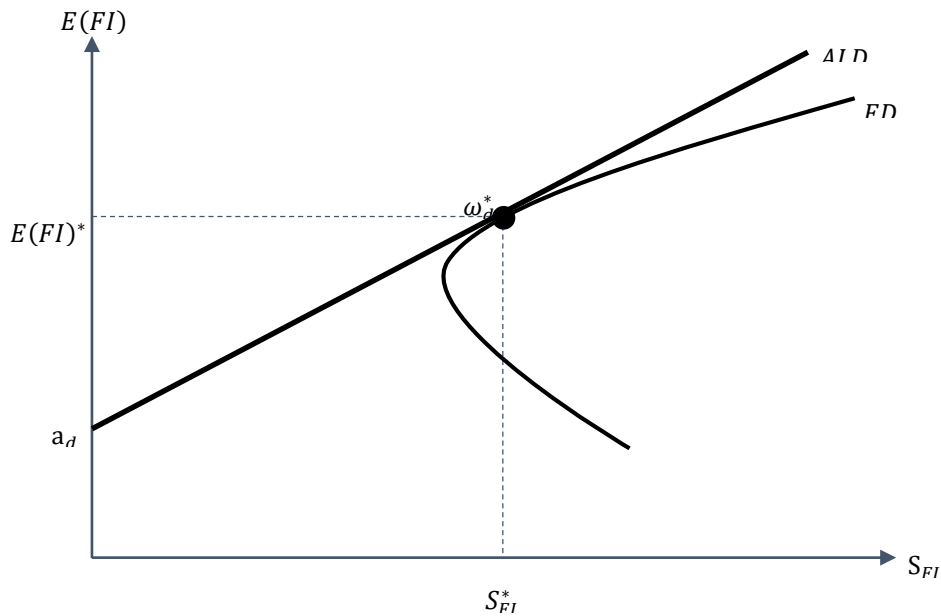


Figure 2. Frontier FD and allocation line ALD of dimensions

### The Process of Research Designment

The research designment includes five basic steps to measure a composite index of financial inclusion.

In the first step, the theoretical framework of financial inclusion and the proposed model depend on the research target of digitalized level for financial inclusion in lower-middle income countries based on the classification of traditional financial inclusion TFI and digital financial inclusion DFI; and suggest an alternative measure of optimal weights for overall financial inclusion FI.

In the second step, the collection of secondary data for indicators of financial inclusion in Table 1 is from the source of FAS-IMF. And processing the raw data of indicators into the research data of normalized indicators

with the range from 0 to 1, then putting these research data into the proposed model to support the two-stage principal component analysis (two-stage PCA).

In the third step, 06 proxy variables of dimensions in the equation (4) estimated in the first-stage PCA are used in estimating optimal weights for overall financial inclusion, based on the combination between expected distance and standard deviation of 06 dimensions to evaluate the importance of dimensions for overall financial inclusion.

In the fourth step, the composite components estimated in the second-stage PCA are used in estimating optimal weights for overall financial inclusion, based on the combination between expected distance and standard deviation of composite components to evaluate the importance of composite components for overall financial inclusion.

In the last step, research results and discussions give contributions to measuring a composite index of financial inclusion and supporting further research.

### Two-Stage Principal Component Analysis (Two-Stage PCA)

The measurement of a composite financial inclusive index in this research is designed with two-stage principal component analysis as follows:

#### The first-stage PCA

Based on 06 dimensions designed in the equation (4), the research indicators designed in Table 1 are used in the first stage principal component analysis.

Table 3. The first-stage principal component analysis of dimensions for financial inclusion

Classification	Dimension	Equation of principal component
Traditional financial inclusion (TFI)	Penetration (TP) Availability (TA) Usage (TU)	$TP = b_{TPDA}TPDA + b_{TPLA}TPLA + b_{TPNC}TPNC + b_{TPND}TPND + \epsilon_{TP}$ $TA = b_{TANB}TANB + b_{TANK}TANK + \epsilon_{TA}$ $TU = b_{TUOD}TUOD + b_{TUOL}TUOL + \epsilon_{TU}$
Digital financial inclusion (DFI)	Penetration (DP) Availability (DA) Usage (DU)	$DP = b_{DPNT}DPNT + b_{DPAO}DPAO + b_{DPAK}DPAK + \epsilon_{DP}$ $DA = b_{DAAO}DAAO + b_{DAAA}DAAA + \epsilon_{DA}$ $DU = b_{DUNT}DUNT + b_{DUVM}DUVM + \epsilon_{DU}$

#### The second-stage PCA

Based on the determinants of financial inclusion through the first-stage principal component analysis, these 06 dimensions are used in the second stage principal component analysis.

Table 4. The second-stage principal component analysis of dimensions for financial inclusion

	Component	Equation of principal component, with $k$ components < 06 dimensions
Traditional financial inclusion (TFI)	TFI <sub>1</sub>	$TFI_1 = b_{1,TP}TP + b_{1,TA}TA + b_{1,TU}TU + \epsilon_{TFI_1}$
	TFI <sub>2</sub>	$TFI_2 = b_{2,TP}TP + b_{2,TA}TA + b_{2,TU}TU + \epsilon_{TFI_2}$
	TFI <sub>3</sub>	$TFI_3 = b_{3,TP}TP + b_{3,TA}TA + b_{3,TU}TU + \epsilon_{TFI_3}$
Digital financial inclusion (DFI)	DFI <sub>1</sub>	$DFI_1 = b_{1,DP}DP + b_{1,DA}DA + b_{1,DU}DU + \epsilon_{DFI_1}$
	DFI <sub>2</sub>	$DFI_2 = b_{2,DP}DP + b_{2,DA}DA + b_{2,DU}DU + \epsilon_{DFI_2}$
	DFI <sub>3</sub>	$DFI_3 = b_{3,DP}DP + b_{3,DA}DA + b_{3,DU}DU + \epsilon_{DFI_3}$

#### Testing PCA

KMO coefficient > 0.5. This is a general coefficient to test reliability of PCA; if KMO<0.5, the PCA findings are not reliable in exploring new component, which is a proxy variable for initial indicators.

Eigenvalue > 1. This is a standard deviation of component; If Eigenvalue<1, the component cannot be a proxy variable for initial indicators.

Total Variance Explained, TVE > 50%. This is a summative coefficient of eigenvalue on total initial indicators that explain the amount of information replaced for total initial indicators; if TVE < 50%, the under-components are quite weak in the explanation of total initial indicators.

Factor Loading, FL > 0.35. This is a correlative coefficient between component and indicator; if significance of FL > 0.05, the component and the indicator are uncorrelated.

### The Estimation of Optimal Weights

The measurement of optimal weights is based on a combination between the expected distance and the standard deviation of 06 dimensions designed in the equation (4) throughout the Lagrange function in the equation (6) for evaluating overall financial inclusion. By taking differentials of parameters  $\omega, v, \mu$  in the equation (6), we have the following equational system:

$$\frac{\partial \mathcal{L}}{\partial \omega} = \omega^T \Sigma_d - [1]^T - [E(d)]^T = 0, \tag{8a}$$

$$\frac{\partial \mathcal{L}}{\partial v} = 1 - \omega^T [1] = 0, \tag{8b}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = E(FI) - \omega^T E(d) = 0, \tag{8c}$$

and solving equational system to find out weights of 06 dimensions in the equation (4). However, seeking optimal weights of 06 dimensions is combined with the Sharpe-ratio in the equation (7) which the Sharpe ratio reaches at the maximized combination between the expected distance and standard deviation of overall financial inclusion.

Similarly, finding out the optimal weights of  $k$  components for overall financial inclusion, variance-covariance matrix of 06 dimensions  $\Sigma_d$  in the equation (6) is replaced into variance-covariance matrix of  $k$  components  $\Sigma_{PC}$ , determined by the following Lagrange function:

$$\mathcal{L}(\omega_{PC}, v_{PC}, \mu_{PC}) = \frac{1}{2} \omega_{PC}^T \Sigma_{PC} \omega_{PC} + v_{PC} [1 - \omega_{PC}^T [1]] + \mu_{PC} [E(FI) - \omega_{PC}^T E(PC)], \tag{9}$$

in which  $\Sigma_{PC}$  is variance-covariance matrix of new components,  $E(PC)$  is expected distance matrix of new components,  $E(FI)$  is expected distance of overall financial inclusion,  $[1]$  is matrix of vector 1 with total weights  $[1]^T \omega = 1$ ,  $\omega$  is weight matrix of new components,  $v_{PC}$  is Lagrange multiplier of the component weights,  $\mu_{PC}$  is Lagrange multiplier of the component expected distances, and symbol  $[...]^T$  is transpose matrix.

### Research Results

The secondary data is collected unstructured data and the analysis of financial inclusion depends on the pooled-data of 55 lower-middle income countries in the period 2004-2022, and the descriptions of actual indicators to give a statistical report as follows:

#### Descriptions

In Table 5, the proposed model has 06 main dimensions, including traditional penetration  $TP$  (4 indicators), digital penetration  $DP$  (3 indicators), traditional availability  $TA$  (2 indicators), digital availability  $DA$  (2 indicators), traditional usage  $TU$  (2 indicators), digital availability  $DU$  (2 indicators), described as follows:

Table 5. Summary statistics of actual indicators



Dimensions	Indicators	Observations	Minimum value	Maximum value	Average value	Standard deviation
Penetration <i>P</i>	Traditional <i>TP</i>					
	TPDA	658	1.23	4984.81	676.35	661.16
	TPLA	570	0.40	853.15	121.19	126.89
	TPNC	308	0.03	470.13	44.35	67.28
	TPND	423	0.69	4945.66	427.11	544.71
	Digital <i>DP</i>					
	DPAK	228	5.00	1275860.00	121526.16	223033.14
Availability <i>A</i>	Traditional <i>TA</i>					
	TANB	964	0.02	86.04	8.96	13.83
	TANK	865	0.01	122.32	14.30	21.70
	Digital <i>DA</i>					
	DAAA	187	0.07	1649.36	255.38	324.71
	DAAO	172	0.17	2750.70	341.63	545.96
Usage <i>U</i>	Traditional <i>TU</i>					
	TUOD	941	4.67	304.40	43.93	30.85
	TUOL	936	0.06	245.23	32.81	26.18
	Digital <i>DU</i>					
	DUNT	264	0.07	239972.45	18873.23	38999.08
	DUVM	251	0.01	180.56	17.43	33.50

Table 1 shows that observations of digital indicators are fewer than the ones of traditional indicators, this also indicates that digitalized level of financial inclusion in lower-middle income countries has happened for 10 years, periods 2015-2024.



Figure 3. Frequency of traditional and digital financial inclusion in lower-medium income countries

Based on formulas (1), (2) and (3), the distance-based financial inclusive dimensional indexes with the weight on each dimension in lower-middle income countries are estimated are shown in Table 6. Among 06 dimensions, two dimensional indexes of digital availability *DA* and traditional usage *TU* reach higher than the others, in which the dimensional index of digital availability is:

$$DA = \frac{1}{2}(X_1 + X_2) = \frac{1}{2}(.137 + .136) \approx .137, \tag{10}$$

Table 6. Average dimensional sub-indexes in the period 2004-2022

Dimensions	Indicators	Variation (CV)	Weight (W)	Point X (d)	Distance {O, X} X1	Distance {X, W} X2	Sub-index $\frac{1}{2}(X1+X2)$
Penetration <i>P</i>	Traditional <i>TP</i>						
	TPDA	0.98	0.203	0.028	0.111	0.108	0.110
	TPLA	1.05	0.217	0.031			
	TPNC	1.52	0.315	0.030			
	TPND	1.28	0.265	0.023			
Digital <i>DP</i>							

	DPAK	1.84	0.392	0.037	0.109	0.107	0.108
	DPAO	1.61	0.343	0.036			
	DPNT	1.25	0.266	0.037			
Availability <i>A</i>	Traditional <i>TA</i>						
	TANB	1.54	0.504	0.052	0.110	0.110	0.110
	TANK	1.52	0.496	0.058			
	Digital <i>DA</i>						
	DAAA	1.27	0.443	0.069	0.137	0.136	0.137
	DAO	1.60	0.557	0.069			
Usage <i>U</i>	Traditional <i>TU</i>						
	TUOD	0.70	0.468	0.061	0.132	0.132	0.132
	TUOL	0.80	0.532	0.071			
	Digital <i>DU</i>						
	DUNT	2.07	0.518	0.041	0.088	0.088	0.088
	DUVM	1.92	0.482	0.047			

Because of normalized weights by the formula (3), the weights of normalized dimensional indexes are the same average in the distance-based measures, that is, dimensional indexes are lack of a combination between the average and the standard deviation for seeking the optimal weights of financial inclusion. Next, the values of normalized indicators for each dimension are used to analyse principal components.

### Two-Stage Principal Component Analysis

In this first-stage PCA, we measure the sub-indices for traditional penetration (TP), digital penetration (DP), traditional availability (TA), digital availability (DA), traditional usage (TU) and digital usage (DU), described in Table 3. The method of PCA find out two key indexes to exploit new components: eigenvalue and eigenvector, in which the eigenvalue, higher than one, represents the standard deviation of new component, and the eigenvector represents the close-coordinated points of normalized multi-indicators.

The results of 06 sub-components in the first-stage PCA is shown that the first principal component on each dimension reaches the conditions of testing PCA:  $KMO > .5$  with significance  $< .05$ ; eigenvalue  $> 1$ ; total variance explained  $> 50\%$ . The first-stage PCA finds out 06 component equations in Table 7, and the expected financial inclusive distance on the sub-indices of 06 dimensions is  $SFI = 0.279$ , where the weight of traditional penetration is the highest ratio  $w_{TP} = 22.4\%$  with its expected distance  $d_{TP} = 0.419$ ; the weight of digital penetration reaches  $w_{DP} = 17.8\%$  with its expected distance  $d_{DP} = 0.291$ ; the weight of digital availability reaches  $w_{DA} = 16.1\%$  with its expected distance  $d_{DA} = 0.261$ .

In the second-stage PCA, we measure two traditional and digital financial inclusive indexes for estimating the expected distance of overall financial inclusion, where the traditional financial inclusive index (TFI) includes three traditional dimensions: traditional penetration (TP), traditional availability (TA), and traditional usage (TU); and the digital financial inclusive index (DFI) includes three digital dimensions: digital penetration (DP), digital availability (DA), and digital usage (DU), shown in Table 7. The results in the second-stage PCA are shown that the expected distance of overall financial inclusive index is  $FI = 0.361$ , in which the weight of traditional financial inclusion occupies 31.7% with its expected distance  $d_{TFI} = 0.254$  and the weight of digital financial inclusion is 68.3% with its expected distance  $d_{DFI} = 0.410$ . These results are shown that the combination between traditional and digital financial inclusion increases the expected distance of overall financial inclusion.

Table 7. Results of two-stage principal component analysis (two-stage PCA)

	Comp.	Eigen.	Diff.	Prop.	Cum.	Component Equation
<b>First-stage PCA</b>						
Penetration <i>P</i>	Traditional <i>TP</i> : $KMO = 0.640$					
	$TP_1$	2.660	1.872	66.50	66.50	$TP = .291TPDA + .355TPLA + .252TPNC + .319TPND$
	$TP_2$	0.788	0.376	19.69	86.19	
	$TP_3$	0.412	0.271	10.29	96.48	
	$TP_4$	0.141		3.52	100.00	
	Digital <i>DP</i> : $KMO = 0.572$					
$DP_1$	2.122	1.406	70.73	70.73	$DP = .313DPAK + .441DPAO + .422DPNT$	

	$DP_2$	0.716	0.554	23.88	94.61	
	$DP_3$	0.162		5.40	100.00	
Availability A	Traditional TA: KMO = 0.500					
	$TA_1$	1.730	1.460	86.48	86.48	$TA = .538TANB + .538TANK$
	$TA_2$	0.270		13.52	100.00	
	Digital DA: KMO = .500					
	$DA_1$	1.907	1.814	95.37	95.37	$DA = .512DAAA + .512DAAO$
	$DA_2$	0.093		4.63	100.00	
Usage U	Traditional TU: KMO = .500					
	$TU_1$	1.864	1.728	93.22	93.22	$TU = .518TUOD + .518TUOL$
	$TU_2$	0.136		6.78	100.00	
	Digital DU: KMO = .500					
	$DU_1$	1.580	1.160	79.01	79.01	$DU = .563DUNT + .563DUVM$
	$DU_2$	0.420		20.99	100.00	
Financial Inclusion on Sub-Indexes	Current average index of 06 dimensions for lower-middle income countries in period 2004-2022 $SFI = \frac{\lambda_i}{\sum_{i=1}^6 \lambda_i} d_i = \frac{2.660}{11.863} 0.419 + \frac{2.122}{11.863} 0.291 + \dots + \frac{1.580}{11.863} 0.161 = 0.279$					
<b>Second-stage PCA</b>						
Traditional Financial Index TFI	KMO = .503					
	$TFI_1$	1.229	0.235	40.97	40.97	$TFI_1 = .632TP + .626TA - .153TU$
	$TFI_2$	0.994	0.217	33.13	74.10	
	$TFI_3$	0.777		25.90	100.00	
Digital Financial Index DFI	KMO = .682					
	$DFI_1$	2.646	2.352	88.19	88.19	$DFI_1 = .358DP + .368DA + .338DU$
	$DFI_2$	0.294	0.234	9.80	97.99	
	$DFI_3$	0.060		2.01	100.00	
Overall Financial Inclusion FI	Current average index of overall financial for lower-middle income countries in period 2004-2022 $FI = \frac{\lambda_{TFI_1}}{\lambda_{TFI_1} + \lambda_{DFI_1}} TFI_1 + \frac{\lambda_{DFI_1}}{\lambda_{TFI_1} + \lambda_{DFI_1}} DFI_1 = \frac{1.229}{1.229 + 2.646} 0.254 + \frac{2.646}{1.229 + 2.646} 0.410 = 0.361$					

### The Sharpe Ratio And Optimal Weight

Finding out an optimal weight for overall financial inclusion is based on the combination between the expected distance and standard deviation to exploit the optimal frontier with tangential point at the maximized Sharpe ratio, shown in the equation (7). In the context of this research, we find out optimal weight on two aspects of dimensions, and traditional-digital inclusive indexes.

#### Optimal weight of dimensions

Based on the equational system (8a), (8b), (8c) we find out the expected distance and the standard deviation of 6 dimensions. Then, the maximized Sharpe-ratio  $R_d^*$  is determined by the following formula:

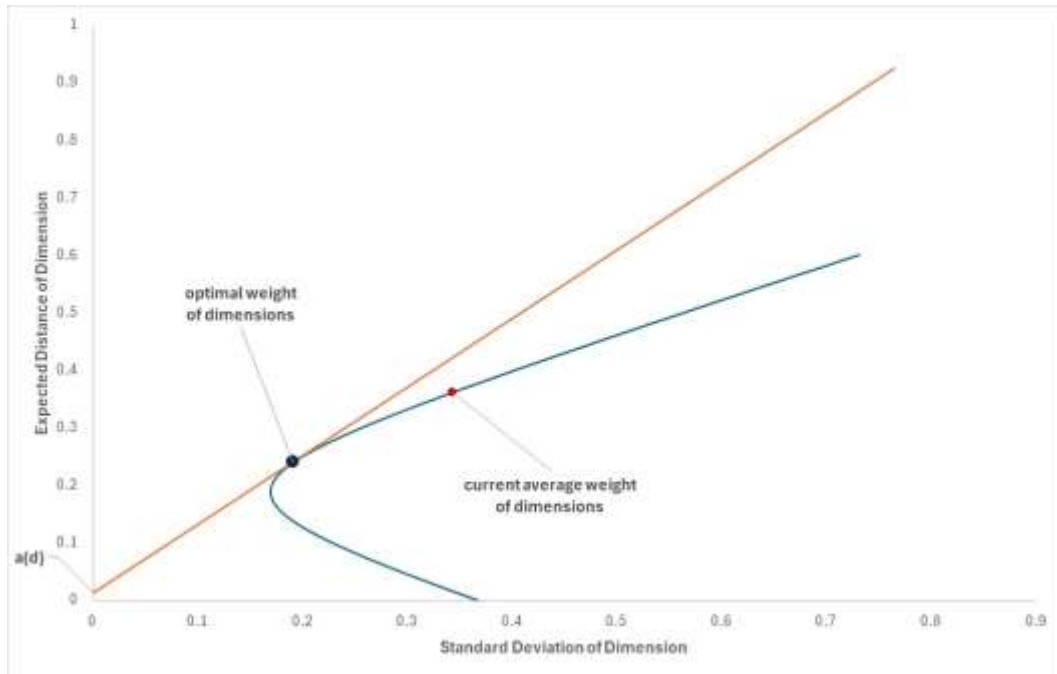
$$R_d^* = \lim_{R_d \rightarrow \max} \frac{[SFI - a_d]}{S_{SFI}} = \frac{[.240 - .012]}{.191} \approx 1.194, \tag{11}$$

where  $a_d$  is stable coefficient of 6 dimensions, calculated by  $a_d = \sum_{i=1}^6 w_i \{ [1 + E(d_i)]^{1/T} - 1 \}$ , to represent the yearly stability of 6 dimensions evaluated in the research sample of lower-middle income countries. At the optimal point of  $R_d^*$ , six optimal weights of dimensions are determined as follows:

$$W_d^* = \{w_{TP}^*, w_{DP}^*, w_{TA}^*, w_{DA}^*, w_{TU}^*, w_{DU}^*\} = \{.224.179.146.161.157.133\}, \tag{12}$$

in which the weight of digital usage is the weakest among six weights of dimensions. Moreover, the stability of digital usage is lower than the stable coefficient of 6 dimensions, that is,  $a_d = .012 > a_{DU} = .007$ . Therefore, the digitalized level of dimensional usage is quite low in lower-middle income countries.

Figure 4. The



combination between expected distance and standard deviation of dimensions

In Figure 4, the current average distance of dimensions reaches  $E(d) = .279$ , higher than the optimal expected distance  $E(d)^* = .240$ , this indicates that the financial systems in lower-middle income countries only pay more attention to the dimension of penetration, rather than the dimensions of availability and usage in overall financial inclusion.

### Optimal Weight of Traditional-Digital Financial Indexes

on the modification of the equational system (8a), (8b), (8c), combined with the equation (9), we find out the expected distance and the standard deviation of two traditional-digital financial indexes. Then, the maximized Sharpe-ratio  $R_{FI}^*$  is determined by the following formula:

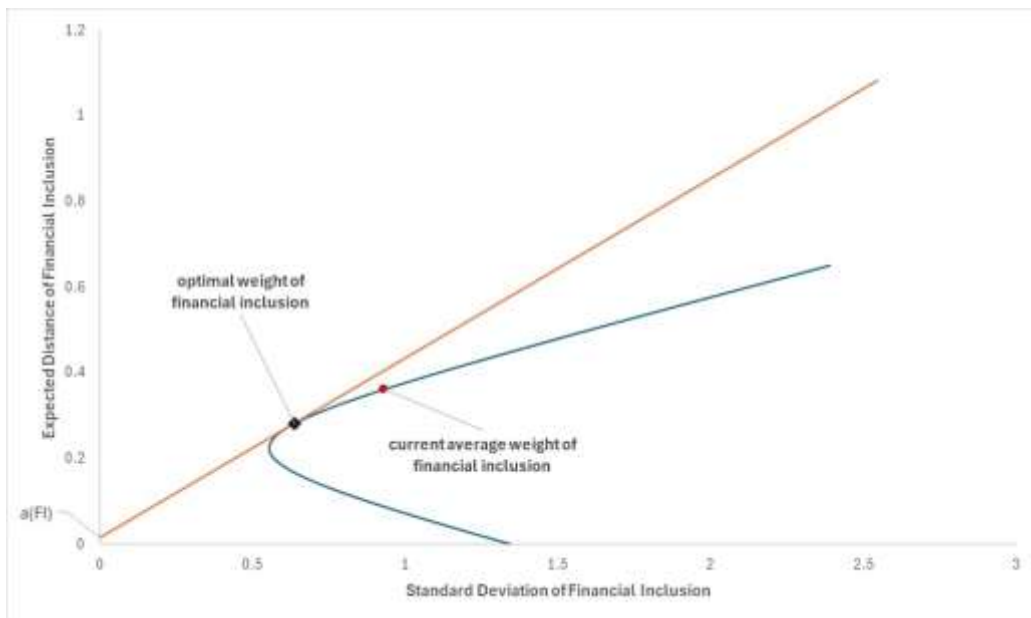


Figure 5. The combination between expected distance and standard deviation of traditional-digital FI indexes

$$R_{FI}^* = \lim_{R_{FI} \rightarrow \max} \frac{[FI - a_{FI}]}{S_{SFI}} = \frac{[.282 - .015]}{.636} \approx 0.420, \quad (13)$$

where  $a_{FI}$  is stable coefficient of traditional-digital FI indexes, calculated by  $a_{FI} = \sum_{i=1}^2 w_i \{ [1 + E(FI_i)]^{1/T} - 1 \}$ , to represent the yearly stability of 2 traditional-digital FI indexes in the research sample of lower-middle income countries. At the optimal point of  $R_{FI}^*$ , two optimal weights of traditional-digital FI indexes are determined as follows:

$$W_d^* = \{w_{TFI}^*, w_{DFI}^*\} = \{.822, .178\}, \quad (14)$$

in which the weight of digital financial inclusion occupies  $w_{DFI}^* = .178$ , to reflect the digitalized level of overall financial inclusion in lower-middle income countries.

## Discussion and Conclusion

This study is based on the methodology of principal component analysis PCA, derived from the recent studies [6; 11; 15; 20; 24] to exploit a new approach of finding out optimal weight in the combination between expected distance and standard deviation, for evaluating the digitalized level of financial inclusion in lower-middle income countries.

The research provides some empirical evidence that the digitalized level of financial inclusion in lower-middle income countries is quite weaker because of the conditions of economic resources and people habitants in access of traditional financial inclusion, rather than digital financial inclusion.

Despite the limitation of composite financial inclusive indexes through the normalized indicators, this leads to biasness of expected distance between indicators, dimensions, traditional-digital FI indexes, the further research needs to develop new indexes that coordinate together in the measurement of overall financial inclusion.

## Conflicts Of Interest

The Authors declare no conflicts of interest.

## REFERENCES

- Abel, S., Mutandwa, L., & Roux, P.L. (2018). A Review of Determinants of Financial Inclusion, *International Journal of Economics and Financial Issues*, 8(3), 1-8.
- AFI (2017). Defining Financial Inclusion, AFI Guideline Note No. 28, 1-8.
- Akakpo, A.A., Amidu, M., Coffie, W., & Abor, J.Y. (2022). Financial Literacy, Financial Inclusion and Participation of Individual on the Ghana Stock Market, *Cogent Economics & Finance*, 10(1), 1-34. doi:10.1080/23322039.2021.2023955
- Amidžić, G., Massara, A., & Mialou, A. (2014). Assessing Countries' Financial Inclusion Standing—A New Composite Index, IMF Working Paper No. 14/36, International Monetary Fund, Washington, D.C.
- Anand, S.K., & Kuldip, S.C. (2013). A Theoretical and Quantitative Analysis of Financial Inclusion and Economic Growth, *Management and Labour Studies*, 38(1&2), 103-133. doi: 10.1177/0258042X13498009
- Banik, B., & Roy, C.K. (2023). Measuring Fintech-Driven Financial Inclusion for Developing Countries: Comprehensive Digital Financial Inclusion Index (CDFII), *Economic Journal of Emerging Markets*, 15(2), 143-159. doi: <https://doi.org/10.20885/ejem.vol15.iss2.art3>
- Cámara, N., & Tuesta, D. (2014). Measuring Financial Inclusion: A Multidimensional Index, Working Paper, No. 14/26, BBVA Research, 1-40.
- Floriant, G. (2023). Nexus between Financial Inclusion and Economic Activity: A Study about Traditional and Non-Traditional Financial Service Indicators Determining Financial Outreach, MPRA Paper No. 119265, <https://mpra.ub.uni-muenchen.de/119265/>
- Goel, S., & Sharma, R. (2017). Developing a Financial Inclusion Index for India, *Information Technology and Quantitative Management*, 122, 949-956. doi: 10.1016/j.procs.2017.11.459
- Gupte, R., Venkataramani, B., & Gupta, D. (2012). Computation of financial inclusion index for India. *Procedia – Social and Behavioral Sciences*, 37, 133-149. doi:

- Hanivan, H., & Nasrudin, N. (2019). A Financial Inclusion Index for Indonesia, *Bulletin of Monetary Economics and Banking*, 22(3), 350-366. doi: <https://doi.org/10.21098/bemp.v22i3.1056>
- Ismael, D.M., & Ali, S.S. (2021). Measuring Digital and Traditional Financial Inclusion in Egypt: A New Index, *International Journal of Applied Research in Management and Economics*, 4(2), 13-34. doi: <https://doi.org/10.33422/ijarme.v4i2.629>
- Khera, P., Ng, S., Ogawa, S., & Sahay, R. (2021). Measuring Digital Financial Inclusion in Emerging Market and Developing Economies: A New Index, *Asian Economic Policy Review*, 17(2), 213-230. doi: <https://doi.org/10.1111/aepr.12377>
- Kim, D.W., Yu, J.S., & Hassan, M.K. (2017). Financial Inclusion and Economic Growth in OIC Countries, *Research in International Business and Finance*, 43, 1-14. doi: <https://doi.org/10.1016/j.ribaf.2017.07.178>
- Lenka, S.K. (2021). Relationship between Financial Inclusion and Financial Development in India: Is There Any Link?, *Journal of Public Affairs*, 22(51), 1-10. doi: [10.1002/pa.2722](https://doi.org/10.1002/pa.2722)
- Loukoianova, E., & Yang, Y. (2018). Financial Inclusion in Asia-Pacific, No. 18/17, International Monetary Fund.
- Lukonga, I. (2018). Fintech, Inclusive Growth and Cyber Risks: Focus on the MENAP and CCA Regions, IMF Working Paper No. 18/201, International Monetary Fund, Washington, D.C.
- Mialou, A., Amidzic, G., & Massara, A. (2017). Assessing Countries' Financial Inclusion Standing – A New Composite Index, *Journal of Banking and Financial Economics*, 2(8), 105-126. <https://www.cceol.com/search/article-detail?id=643150>
- Ozili, P.K. (2022). Digital Financial Inclusion, MPRA Paper No. 113789, 1-15. <https://mpa.ub.uni-muenchen.de/113789/>
- Park, C.-Y., & Mercado, R. V. Jr. (2018). Financial Inclusion: New Measurement and Cross-Country Impact Assessment, Working Paper Series No. 539, ADB Economics
- Pesqué-Cela, V., Tian, L., Luo, D., Tobin, D., & Kling, G. (2021). Defining and Measuring Financial Inclusion: A Systematic Review and Confirmatory Factor Analysis, *Journal of International Development*, 33(2), 316-341. doi: <https://doi.org/10.1002/jid.3524>
- Peterson, O. (2023). Impact of Financial Inclusion on Economic Growth: Review of Existing Literature and Directions for Future Research, MPRA Paper No. 118788, 1-26.
- Rapih, S., & Wahyono, B. (2023). The Relative Impact of Traditional and Digital Financial Inclusion on Economic Growth: A Threshold Regression-Based Comparative Analysis, *International Review of Applied Economics*, 37(6), 1-23.
- Ren, K., Wang, Y., & Liu, L. (2023). Impact of Traditional and Digital Financial Inclusion on Enterprise Innovation: Evidence from China, *SAGE Open*, 1-19. doi: <https://doi.org/10.1177/21582440221148097>
- Sarma, M. (2008). Index of Financial Inclusion, Working Papers. No.215, Indian Council for Research on International Economic Relations, New Delhi.
- Sarma, M. (2015). Measuring Financial Inclusion, *Economics Bulletin*, 35(1), 604-611.
- Tram Thi Xuan Huong, Lai Tien Dinh, & Nguyen Thi Truc Huong (2021). Constructing A Composite Financial Inclusion Index for Developing Economies, *Quarterly Review of Economics and Finance*, 87, 257-265. doi: <https://doi.org/10.1016/j.qref.2021.01.003>
- Zins, A., & Weill, L. (2016). The determinants of financial inclusion in Africa, *Review of Development Finance*, 6(1), 1-12. <https://journals.co.za/doi/abs/10.1016/j.rdf.2016.05.001>