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# Exploring The Didactic Transposition of The Metric Space

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***Abstract***

*The didactic transposition is used to adapt scientific knowledge into taught knowledge for students through two steps, namely external and internal transposition. This process is crucial when applied to mathematics content as it enhances the understanding of complex concepts and makes knowledge more accessible to learners. To create effective lessons and scholarly knowledge objects, teachers must establish connections between their learned knowledge and the lesson structure. Metric and probabilistic metric spaces are two complex structures of knowledge that require understanding and teaching. This article explores the didactic transposition approach for teaching the concept of metric space, including probabilistic metric space, through external and internal steps of transposition, it provides two examples of transposing the Metric Distance notion in the Euclidean space, as well as two examples of the probabilistic metric space, aiming to transpose function distribution distance. It also outlines strategies for introducing metric spaces to students, referencing examples while acknowledging the teaching obstacles posed by this concept.*

***Keywords:*** *Metric Spaces, Didactic Transposition, Mathematics Distance*

## INTRODUCTION

The concept of didactic transposition involves the adaptation, transformation and transposition of knowledge and knowledge objects (Bosch & Gascón, 2006; Chevallard & Bosch, 2014; Topphol, 2023). This concept was first introduced by Chevallard in 1980 during the first summer school on mathematics didactics. It followed Michael Verret's initial notion of didactic transposition in 1975. Since then, it has been used in educational disciplines (Achtoun et al., s. d.; Bosch & Gascón, 2006; Chevallard et al., 1985; Mercier, 2002). According to Chevallard, didactic transposition is crucial in providing efficient learning and teaching. It involves programmable teaching and is the transition from learned knowledge to taught knowledge.(Chevallard et al., 1985).

On the other hand, M. Verret outlines five steps for applying didactic transposition from scholarly objects to learned knowledge (Perrenoud, 1998). The first step is to desynchronize knowledge by structuring it into different fields and domains. Although scholarly knowledge is already organized into disciplines, there is no equivalent for other human knowledge. It is important to perform this task objectively and without bias. Secondly, depersonalizing knowledge is crucial to ensure that it is not tied to individuals. Additionally, programming knowledge involves organising and sequencing learning activities in a chosen order, typically in order of chronological complexity. Moreover, advertising knowledge culminates in reference frameworks and curricula that enable everyone to understand the intended subject matter. Lastly, the final step is controlling the acquisition.

When approaching the process of didactic transposition, didacticians may face several problems. One of these is the significant gap between scholarly and taught knowledge. The transposition aims to reduce this gap as much as possible for two main reasons. Firstly, as Chevallard states, 'no taught knowledge would know how to authorize itself', meaning that the existence of learned knowledge necessitates the existence of school knowledge (Achtoun et al., s. d.; Artigue, 1991). This knowledge epistemologically guarantees school

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knowledge in the eyes of society. Additionally, while the mathematics taught may not be identical to that of mathematicians, it must still enable students to later understand the mathematics of mathematicians."

The process of transposition can be divided into two stages: external transposition and internal transposition. External transposition involves transforming scholarly knowledge into knowledge that can be taught, while taking into account the social practices of reference. Internal transposition is a continuous process that involves transforming knowledge that can be taught into knowledge that is taught, and then into knowledge that is acquired. To ensure effective external transposition, certain rules must be respected. The role of internal transposition is to transform the knowledge to be taught into knowledge that is taught and then acquired by the learner. The transposition process is illustrated in the diagram of Figure 1

The didactic transposition diagram indicates that the probabilistic metric space represents the knowledge acquired in this study, which was initially proposed by K. Menger in 1942. (Achtoun et al., s. d.). The role of the designer is the external transposition which consists of transforming and reorganising this learned knowledge into knowledge to be taught (manuals or programmes). In this context, they will reorganise the concept of probabilistic metric spaces for the context of school teaching. The taught knowledge is the concept of probabilistic metric spaces taught by the teacher to the learners under a set of conditions (prerequisites, difficulties, means, etc.).

Finally, the acquired knowledge is the concept of probabilistic metric spaces learned and designed by students. To interpret school mathematical knowledge, it is necessary to consider the phenomena related to the reconstruction of school mathematical knowledge (Achtoun et al., s. d.; Bosch & Gascón, 2014) from the learned mathematical knowledge, in which external transposition plays an essential role in mathematics education. Thus, internal transposition is a crucial process for transitioning mathematical teaching programs and manuals to appropriate learning sequences. This research aims to develop and analyze the form of internal and external transition of the concept of probabilistic metric spaces from learned knowledge to taught knowledge.

In mathematics education, the challenge is to adapt learned mathematical knowledge into teachable content. The Didactic Transposition approach has been developed to address this challenge (Achtoun et al., s. d.; Bosch & Gascón, 2006; Chevallard et al., 1985). This paper aims to apply the Didactic Transposition Approach to move the notion of Metric Space and Probabilistic Metric Space from learned knowledge to scholarly knowledge, using both external and internal methods of didactic transposition. Basic strategies for introducing metric spaces to students are also outlined, with reference to concrete examples, and with an awareness of the barriers to teaching this concept. In addition to responding to the following question:

What are the difficulties of the application of Didactic transposition approach for the Metric and Probabilistic Metric spaces concept?

## Problem of the Notion of Distance

The mathematical concept of distance is a fundamental notion that permeates various branches of mathematics and has profound implications in numerous scientific disciplines. The classical definition of distance in a Euclidean space is deeply rooted in geometric intuition, where the distance between two points is determined by the length of the straight line segment connecting them, a concept that can be rigorously established using the Pythagorean theorem. This Euclidean metric is but one example of a broader class of functions known as distance functions or metrics, which are defined on a set and satisfy certain axioms—namely, non-negativity, identity of indiscernible, symmetry, and triangle inequality.

The study of these functions is central to the field of metric space theory, as articulated by David Hartenstine in 2004. In a metric space, the distance between elements is not necessarily derived from a physical measurement but is an abstraction that retains the essential properties of the Euclidean notion of distance. This abstraction allows mathematicians to generalize the concept of distance to encompass a wide variety of mathematical objects and structures, thereby enabling the analysis of spaces that may not have a direct geometric or physical interpretation.

The evolution of the concept of distance has been particularly significant in the context of topology. Initially developed in the realm of the real line, the topological notion of distance was extended to more complex structures, such as functional spaces, which often possess infinite dimensions. The topological approach to distance focuses on the continuity of functions and the convergence of sequences, providing a framework for understanding the proximity and continuity of mathematical entities without relying on a specific metric.

One of the pivotal motivations behind the generalization of distance is the desire to quantify and analyze the similarity or dissimilarity between objects. This is not limited to points in a geometric space but can also apply to functions, shapes, probability distributions, and other mathematical constructs. The notion of similarity captured by a distance function is not always intuitive, as it may not correspond to the conventional geometric understanding of closeness. For example, in function spaces, two functions may be considered close if they behave similarly within a certain context, even if they differ significantly at certain points.

This divergence between the mathematical abstraction of distance and the everyday understanding of the term presents a challenge in educational settings. The scholarly knowledge of distance, rich in its complexity and nuance, often stands in stark contrast to the more simplistic representations typically introduced in school curricula. To bridge this gap, educators employ the process of didactic transposition, whereby the scholarly knowledge is transformed into school knowledge—knowledge that is structured and adapted to facilitate student learning.

The process of internal didactic transposition involves selecting and simplifying mathematical concepts to make them accessible to learners at various educational levels. It is a delicate balance to maintain the integrity of the mathematical ideas while ensuring that they are comprehensible and relevant to students. In the case of distance, this may involve starting with concrete examples and gradually introducing more abstract notions, or it may entail the use of visual aids, analogies, and interactive activities to convey the essence of the concept.

Furthermore, the mathematical concept of distance extends beyond the confines of pure mathematics and finds applications in physics, computer science, and engineering. In these disciplines, distance functions are tailored to specific contexts, such as measuring the dissimilarity between data points in machine learning algorithms or assessing the efficiency of network routing protocols in computer networks.

The mathematical concept of distance is a rich and versatile notion that has evolved significantly from its geometric origins. Its applicability across various domains of mathematics and its relevance to other scientific fields underscore the importance of a nuanced understanding of distance. As educators navigate the complexities of conveying this concept to students, they play a crucial role in shaping the learners' appreciation for the depth and breadth of mathematical ideas.

## METHODS AND RESULTS

### Didactic Transposition of Metric Space

Didactic transposition is an essential concept in mathematics didactics that explores how mathematical concepts are taught and learned. When it comes to advanced mathematical concepts such as metric space, it is particularly important to understand how teachers and students interact with these complex ideas. Metric space is a fundamental mathematical structure that defines the notion of distance between points in a set.

The process of didactic transposition involves the translation of mathematical knowledge from an expert domain into a language and methods that are accessible to students. Moreover, in the initial stage of this process, expert mathematical knowledge is transferred externally by experts in various roles, including inspectors, pedagogical engineers, scientific curriculum designers, and others, to export the knowledge to be taught from the scientific domain. In the second instance of the didactic transposition process, the concept is transposed from the teacher to the student, to be taught and learned. (Bosch, s. d.). In addition, the content to be taught is dependent on a number of factors, as highlighted by Prrenoud (1998). These include social needs, specific intelligences, the individual needs of the student, and others (Do & Nguyen, 2020).

When it comes to teaching metric space, this can be a challenge due to this field's abstract and conceptual nature. Teachers need to find ways of making these concepts more concrete and comprehensible to students.

In this part, we applied the external and internal didactic transposition in two mathematics examples for transposing the metric space notion.

## External Transposition

*Scholarly knowledge*

A metric space is a mathematical structure consisting of a set and a function called "distance", which assigns to each pair of elements of this set a non-negative real number satisfying certain properties. Formally, let X be a set, a function $d : X \times X \rightarrow IR$ is a distance on X if it verifies the following three properties for any $x, y, z \in X$ , called the distance axioms:

Positivity: if $d(x, y) \geq 0$ and $d(x, y) = 0$ only if $x=y$ .

Symmetry: $d(x, y) = d(y, x)$ for all $x, y \in X$.

Triangular inequality: $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$

The set X associated with the distance function d is called a metric space, denoted (X,d). The distance d(x,y) is often interpreted as the "distance" between points x and y in this metric space. Metric spaces are an important generalization that allows the notion of distance to be studied in abstract mathematical contexts, and they are widely used in topology, analysis, and other branches of mathematics. Classic examples of metric spaces include Euclidean space with Euclidean distance, but many others can have different and interesting properties. Metric spaces are used to study the concepts of convergence, continuity, compactness, and other important properties in various branches of mathematics, including real analysis and topology.

*Knowledge to be taught*

*Formal Curriculum (Objectives): General guidelines for teaching this concept.*

Several strategies can be used to make the concepts and properties of metric spaces more accessible to students. One of the main strategies is to use concrete examples, teachers can start with familiar examples to introduce the concepts of metric spaces, for example, distances between objects in physical space or distances on a map can be used to illustrate the concept of distance in a metric space. I addition to that, the use of graphical representations, such as charts, diagrams, or geometric figures, can assist students in visualizing the concepts of metric spaces, for example, a teacher can use a graphical representation to illustrate the distance between two points on a Cartesian plane, which can facilitate comprehension. On the other hand, Dynamic geometry software and computer simulations can be integrated to help students interactively explore and experience the properties of metric spaces. These tools also allow for the manipulation of complex and abstract examples. Additionally, teachers can make mathematical concepts more accessible to students by using more intuitive notations or familiar terms instead of abstract mathematical symbols. Also, Encouraging students to solve problems related to metric spaces can help them apply the concepts studied and reinforce their understanding, and the problems can be practical or abstract and can involve real-life situations or mathematical contexts. Furthermore, teachers can encourage dialogue and Q&A sessions to facilitate student questions, ideas, and discussions about metric space concepts. This promotes active engagement among students and clarifies any points of confusion or difficulty.

To introduce the concepts of metric spaces, here are some commonly used concrete examples:

> Distance between two points in a plane: A common example is to use a Cartesian plane and calculate the distance between two points using the Euclidean distance formula. For example, students can measure the distance between two cities on a map using latitude-longitude coordinates.
>
> Distance between two cities: Distances between cities on a map are often used as a concrete example to illustrate distance concepts in metric spaces. Students can calculate the distance between two cities

using methods such as great circle distance (the shortest distance between two points on a sphere) or road distance.

Distance between objects in physical space: Teachers can use concrete examples of objects in physical space to illustrate the notion of distance in metric space. For example, measure the distance between two trees in a forest, or between two buildings in a city.

Hamming distance: Hamming distance is a common example used to illustrate the distance between two strings of the same length. It counts the number of positions where the characters differ. For example, compare two words such as "cat" and "dog" by counting the number of letters that differ.

Manhattan distance: Manhattan distance is used to represent the distance between two points in a plane by calculating the sum of the absolute differences in the coordinates of the points. This can be illustrated using the streets of a city, where the distance between two points is given by the number of blocks tonbe covered horizontally and vertically.

Distance between colors: In the field of color vision, distances between colors can be used as a concrete example of metric space. For example, the Euclidean distance in RGB space can be used to calculate the difference between two colors.

*The Difficulties of Teaching Metric Space*

When teaching metric space, teachers face several challenges. They must:

Make the notion of distance understandable: Explain what a metric is, how it works and why it is important.

Illustrate abstract concepts: Metric spaces often involve abstract concepts such as convergence and continuity. Teachers need to find concrete examples and visual representations to help students grasp these ideas.

Provide relevant exercises: Students need to practice solving problems related to metric space to reinforce their understanding. Teachers need to design appropriate exercises.

## Internal Transposition

*Actual Curriculum (Content): Knowledge Taught*

Activity 1:

the plane (P) is referred to a direct orthonormal reference point R( O; $\vec{\imath}$; $\vec{\jmath}$)

$\vec{u} = x\vec{\imath} + y\vec{\jmath}$ and $\vec{v} = x'\vec{\imath} + y'\vec{\jmath}$ two vectors of the plane as presented in figure 2.

1) Calculate: $\vec{u}.\vec{v}$ as a function of x , y , x' and y'.

2) Calculate: $\vec{u}.\vec{u}$ as a function of x , y .

3) Deduct the norm of $\vec{u}$.

4) Calculate the distance AB as a function of the coordinates of A($x_A, y_A$) and B($x_B, y_B$)

<u>N.B:</u> we note that $\vec{u}.\vec{v} = xx' + yy'$ is correct just in the orthonormal reference point.

Activity 2:

the plane (P) is referred to a direct orthonormal reference point R( O; $\vec{\imath}$; $\vec{\jmath}$)

be two points A($x_A, y_A$) and B($x_B, y_B$)

1) that represents $x_B\_x_A$.

2) that represents$y_B_y_A$

3) Applying the Pythagorean theorem deduce AB² as function of $(x_B\_x_A)^2$ and $(y_B\_y_A)^2$.

4) Deduce the distance AB as a function of the coordinates of A$(x_A, y_A)$ and B$(x_B, y_B)$

## Function Distribution Distance

This section concerns an example of the external transposition of probabilistic metric spaces into educational contexts. To accomplish this, we suggest using the Euclidean space with the Euclidean metric (Euclidean distance) as an example of a metric space that has already been transposed into educational contexts. To introduce the concept of a probabilistic metric space, we substituted the distance with a distribution distance function. During the development of this function, our aim was to adapt it to the cognitive styles of the learners (Achtoun et al., s. d.) to facilitate their learning and ensure comprehension. An example of a didactic sequence suitable for textbooks is provided to illustrate the internal transposition, along with a summary of the transposition process.

To externally transpose the concept of probabilistic metric space, we replaced the Euclidean metric with a carefully selected distance distribution function in a Euclidean space, as outlined in the Figure 4.

And for internal transposition, we propose simple activities to help learners construct this concept.

## External Transposition

*Scholarly knowledge*

Here, we introduce the concept of a probabilistic semi-metric space. Before doing so, let us first return to the concept of a distance distribution function.

Definition 1.(Achtoun et al., s. d.; Fatkic, s. d.)

Let $\Delta^+$ be the space of all distance distribution functions

$f: [0, +\infty] \to [0, 1]$

Such that:

1- $f$ is left continuous on $[0, +\infty]$,

2- $f$ is non-decreasing.

3- $f(0) = 0$ and $f(+\infty) = 1$.

The subset $D^+ \subset \Delta^+$ is the set $D^+ = \left\{f \in \Delta^+ : \lim_{x\to\infty} f(x) = 1\right\}$

As a special element of $D^+$ is the function $\varepsilon_0$ defined by:

$$\varepsilon_0(x) = \begin{cases} 0 & if \quad x \leq 0 \\ 1 & if \quad x > 0 \end{cases}$$

Definition 2.(Achtoun et al., s. d.; Fatkic, s. d.)

A Probabilistic semi-metric space is a couple $(X, F)$ where $X$ is nonempty set, $F$ is a function from $X \times X$ into $\Delta^+$, and the following conditions are satisfied for all $p, q \in X$:

1- $F_{p,p} = \varepsilon_0$

2- $F_{p,q} \neq \varepsilon_0 \; if \; p \neq q$

3- $F_{p,q} = F_{q,p}$

## DISCUSSION

of learned mathematics on school mathematics necessitates adaptation of this knowledge, which involves modifying its structure, sequence, form, and context. To achieve this, several strategies and techniques can be employed to apply the didactic transposition method to mathematical concepts, making them teachable. On the other hand, while implementing these strategies, there are several challenges in the process of translating mathematical knowledge into a more comprehensible form for learners. These challenges include selecting appropriate illustrations that accurately represent the original concept, designing exercises and specific problem situations that allow learners to practice and build their new knowledge, which is referred to as 'Acquired Knowledge'.

In the previous sections, we transposed the form, structure, and context. To introduce the notion of metric space, we chose two examples familiar to learners, and for generalisation, we chose two others to introduce the concept of probabilistic metric space. Next, we will cover the entire process chain by dividing this discussion into stages of transposition.

## External Transposition

Didacticians confirm that external didactic transposition is a process of decontextualisation followed by a recontextualisation of scholarly knowledge to be taught. This process involves taking scholarly knowledge out of its initial epistemological context and placing it in a pedagogical context.

The concept of transposed scholarly knowledge was validated and labelled by the scientific community around 1942. Paun (Achtoun et al., s. d.)suggests that scholarly knowledge can be transformed through simplification, the introduction of terminological equivalents, and the use of figurative language.

*Simplification*

This is a simplified representation of the scientific reference model. The simplification must respect both the basic scientific meaning and the conceptual identity of the scientific referent.

To achieve this, we have introduced a simplification that minimizes distortion, as demonstrated by the four examples we have chosen.

*Terminological transposition*

The terminological transposition terms aim to find equivalent terms to make the content more accessible for learning. This process should be done without altering the scientific concepts' meaning.

*Figurative aspects*

The use of figurative language in educational texts can help students to better understand abstract concepts found in scientific texts. This is demonstrated by the figures and landmarks chosen for the activities.

## Internal Transposition

This relationship is personalized, ideologized, axiologised, and sociologised. The term 'intern' refers to the transposition that occurs within the teacher-student relationship, which constitutes the objectification of differences between them and the formal curriculum. The internal didactic transposition can be considered a process of specification and new curricular meaning.

To understand the changes regarding internal transposition, we must examine the framework of analyses concerning the didactic contract that describes the cognitive and socio-affective action and reciprocal relationship between the teacher and their student (Bosch, s. d.; Do & Nguyen, 2020).

The substance of internal didactic transposition is materialised by two types of curriculum.

The 'real curriculum' or 'taught knowledge' refers to the new concepts introduced by the teacher and learned by the student. In this case, these concepts include metric spaces and the function of distance distribution. The student constructs their understanding of these concepts using a distance between two points, a simple function, and a geometric and functional framework for the general notion of a metric probabilistic space (Bosch et al., 2021).

The realized curriculum, also known as 'the learned and retained knowledge' according to Chevallard (Chevallard, 1985; Chevallard & Bosch, 2014), represents a personalized curriculum that expresses the student's particular relationship to the taught concept. The objective is to challenge the cognitive and sociocultural conceptions of the students regarding the notion of metric space and distance and to introduce a more general notion, namely the metric space and the function distribution distance.

## CONCLUSION

Mathematical knowledge is in a constant state of evolution, giving rise to the emergence of new objects, knowledge, and tools that can impact the fields of science and engineering. It is therefore important to minimise the gap between scholarly and taught knowledge, which is the role of the didactic transposition approach. Furthermore, the process of didactic transposition from the external to the internal steps provides an opportunity to integrate several modern changes to the material to be taught, including the use of AI approaches, Metaverse techniques, and augmented learning, which is based on augmented reality and virtual reality.(Martínez et al., 2017; Mouali et al., 2024).

The examples presented in this work are limited to metric and semi-metric spaces. Two examples of didactic transposition of the metric distance concept are presented. In future work, we will provide additional examples of the transposition of probabilistic metric spaces by approaching the triangular norm and improving certain aspects related to construction, acquisition, and sharing. Furthermore, it is possible to create a tool of didactic transposition based on augmented and virtual realities that can facilitate the external and internal parts of the process.

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